Globaliojo optimizavimo algoritmų, nereikalaujančių išvestinių informacijos, kūrimas, tobulinimas ir realizacija

Doktoranto LINO STRIPINIO ataskaita už 2019/2020 mokslo metus

Darbo vadovas prof. dr. Remigijus Paulavičius

Doktorantūros pradžios ir pabaigos metai: 2016 – 2020

Ataskaitos turinys

Tyrimo objektas ir tikslai

Tyrimo objektas:

- išvestinių informacijos nereikalaujantys DIRECT-tipo globalios optimizacijos algoritmai uždaviniams su įvairaus tipo ribojimais;
- lygiagrečių kompiuterinių sistemų taikymas globaliam optimizavimui;

Tyrimo tikslai:

- tobulinti ir modifkuoti esamus globaliojo optimizavimo algoritmus, kurie nereikalauja išvestinių informacijos, siekiant greičiau ir tiksliau spręsti optimizavimo uždavinius;
- panaudoti lygiagrečias kompiuterių sistemas ir lygiagrečias globaliojo optimizavimo algoritmų versijas spręsti didesnius optimizavimo uždavinius;
- sukurtais algoritmais spręsti praktinius uždavinius.

Planuojami rezultatai

Planuojami rezultatai:

- apžvelgti esamus globaliojo optimizavimo algoritmus, kurie nereikalauja išvestinių informacijos ir apsibrėžti tiriamų globaliojo optimizavimo algoritmų grupę;
- patobulinti esamus ir pasiūlyti naujus apibrėžtos klasės globaliojo optimizavimo algoritmus;
- gautus rezultatus palyginti su rezultatais, gautais taikant jau žinomus globaliojo optimizavimo algoritmus;
- pasiūlyti lygiagrečių globaliojo optimizavimo algoritmų versijas;
- pritaikyti sukurtus algoritmus praktiniams uždaviniams.

2019/2020 m. m. darbo planas

2019/2020 m. m. darbo planas:

- DIRECT algoritmo tobulinimas uždaviniams su paslėptais ribojimais;
- 1 eksperimentinio tyrimo mokslinis straipsnis periodiniame leidinyje;
- Disertacijos rengimas.

Gauti moksliniai rezultatai

DIRECT algoritmo analizė:

- Ištirtos *DIRECT* algoritmo modifikacijos uždaviniams su paslėptais ribojimais;
- Sukurtas *DIRECT-GLh* algoritmas uždaviniams su paslėptais ribojimais;
- Atlikta lyginamoji analizė su visomis esamomis DIRECT modifikacijomis uždaviniams su paslėptais ribojimais.

Existing *DIRECT*-type algorithms for problems with hidden constraints

Algorithm	Strategy
DIRECT-Barrier [1]	*DIRECT* extension based on a barrier approach, which simply assigns a very high value to a hyper-rectangle with infeasible point.
DIRECT-NAS [1]	*DIRECT* extension based on a neighbourhood assignment strategy (NAS), which assigns the value to an hyper-rectangle with infeasible point based on feasible objective function values found in the neighbourhood.
DIRECT-sub-div [2]	A modified *DIRECT-Barrier* algorithm with a sub-dividing step to handle hidden constraints. Sub-dividing step is performed only in specific iterations where all infeasible hyper-rectangles are identified as potential optimal and will be divided together with potential optimal hyper-rectangles obtained after the selection step.

[1] J.M. Gablonsky (2001).
Modifications of the DIRECT algorithm.
Ph.D. thesis, North Carolina State University;

[2] J. Na, Y. Lim, C. Han (2017).
A modified DIRECT algorithm for hidden constraints in an LNG process optimization.
Energy p. 488?500 (2017). DOI 10.1016/j.energy.2017.03.047;

A new *DIRECT-GLh* algorithm for problems with hidden constraints

PHASE 1: Finding a feasible point

Like original *DIRECT-GL*, *DIRECT-GLh* starts from transforming a feasible region D into the unit hyper-cube $\bar{D} = [0,1]^n$. Let the $\mathbb{I}_k$ is the index set identifying the current iteration k set of hyper-rectangles and δ^i is a measure (distance, size) of the hyper-rectangle $\bar{D}^i$.

The uniform partitioning of $\bar{D}$ is performed using **Definition 1**:

Definition (1)

Consider $\bar{D}$, set $\mathbb{I}_k = \{1\}$, where $k = 1$, and perform the following steps:

- **Step 1** Find an index $j \in \mathbb{I}_k$ and a corresponding hyper-rectangle $\bar{D}^j$, such that

$$\bar{D}^j = \arg\max_j \left\{ j = \arg\max_{i \in \mathbb{I}_k} \left\{ \delta^i \right\} \right\}. \quad (1)$$

- **Step 2** Subdivide (trisect) a hyper-rectangle $\bar{D}^j$ and check the feasibility at midpoints of all new hyper-rectangles. Also, set $k = k + 1$, update $\mathbb{I}_k$, and all measures δ^{j_i} of new hyper-rectangles.
- **Step 3** If $D^{\text{feas}} = \emptyset$ **repeat** from Step 1; otherwise **terminate**.

A new *DIRECT-GLh* algorithm for problems with hidden constraints

PHASE 2: Improving a feasible solution

The new design strategy, for hyper-rectangles with infeasible midpoints assigns a value depending on how far that point is from the current best minima

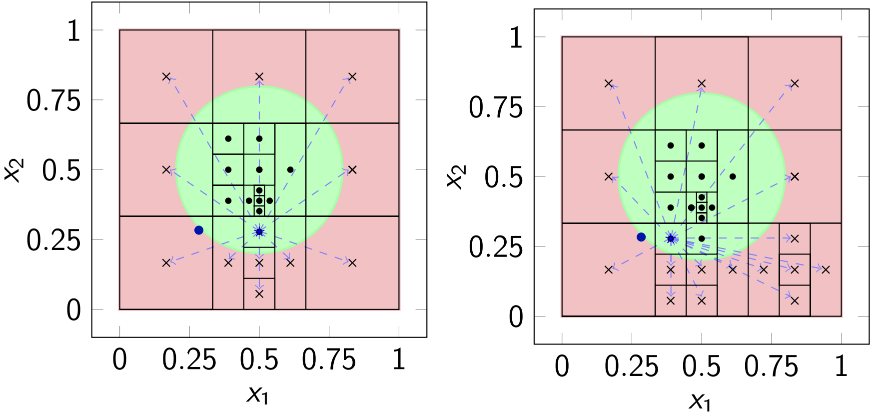
$$\min_{\mathbf{x} \in D} \xi(\mathbf{x}, \mathbf{x}^{\min}, f^{\min}),$$
$$\xi(\mathbf{x}, \mathbf{x}^{\min}, f^{\min}) = \begin{cases} f(\mathbf{x}), & \text{if } \mathbf{x} \in D^{\text{feas}} \\ f^{\min} + d(\mathbf{x}^{\min}, \mathbf{x}), & \text{otherwise}, \end{cases} \tag{2}$$

where $f^{\min}$ is the best feasible objective function value found so far, and $d(\mathbf{x}^{\min}, \mathbf{x})$ is Euclidean distance from the current best minimum point $(\mathbf{x}^{\min})$ to the point $(\mathbf{x})$:

$$d(\mathbf{x}^{\min}, \mathbf{x}) = \sqrt{\sum_{j=1}^{n} \left(x_j^{\min} - x_j \right)^2}. \tag{3}$$

A new *DIRECT-GLh* algorithm for problems with hidden constraints

Geometric illustration of *DIRECT-GLh* on two-dimensional *T1* test problem in the third (left side), and in the fourth (right side) iterations.



$\star$ Current minimum point $\mathbf{x}^{\min}$ ● Feasible points Infeasible region Feasible region

× Infeasible points – – → Distances $d(\mathbf{x}^{\min}, \mathbf{x})$ ● Global minimum point $\mathbf{x}^*$

Experimental investigation

Test problems from the most recent version of the library *DIRECTLib* [3] (67 in total) are used to evaluate the performance of algorithms. It is assumed that any information about the constraint functions is unavailable. In the experimental setup, the hidden searching area D^{hidden} is implemented as:

$$D^{\mathrm{hidden}} = \{\mathbf{x} \in D : \mathbf{g}(\mathbf{x}) \leq 0, \mathbf{h}(\mathbf{x}) = 0\}, \tag{4}$$

but this information is used only to determine whether a certain point is feasible or not. Since global minima f^* are known for all collected test problems, tested algorithms were stopped either when a point $\mathbf{x}$ was generated such that the percent error (pe):

$$pe = 100 \times \begin{cases} \frac{f(\mathbf{x}) - f^*}{|f^*|}, & f^* \neq 0, \\ f(\mathbf{x}), & f^* = 0, \end{cases} \tag{5}$$

is smaller than the tolerance value ($\varepsilon_{\mathrm{pe}} = 10^{-2}$), i.e., $pe \leq \varepsilon_{\mathrm{pe}}$, or when the number of function evaluations exceeds the prescribed limit of 10^6. Additionally, the maximal time for solving one test problem was restricted to six hours.

[3] L. Stripinis, R. Paulavičius (2020).
DIRECTLib – a library of global optimization problems for DIRECT-type methods.
v1.2, DOI 10.5281/zenodo.3948890. URL https://doi.org/10.5281/zenodo.3948890;

Experimental investigation

Group 1: case then initial sampling points are infeasible

The total number of function evaluations needed by algorithms to find the first feasible point ($\mathbf{x} \in D^{\text{feas}}$) on a selected subset of test problems from *DIRECTLib*

Label	n	Constr. type	DIRECT-GLH	DIRECT-NAS	DIRECT-Barrier	DIRECT-sub-div ($\lambda = 2$)	DIRECT-sub-div ($\lambda = 5$)
Bunnag 6	10	L	**136,549**	363,929	1,240,029	5,614,449	1,951,629
G06	2	NL	**33,211**	39,063	59,049	202,589	136,125
G16	5	NL	**803**	2,513	2,673	39,253	25,203
Genocop 9	4	L	8,939	**7,543**	10,935	42,261	53,695
Genocop 11	4	L	**2,889**	7,917	9,477	73,881	36,549
hs021mod	7	L	**33,051**	33,309	85,293	127,569	54,435
P2d	5	NL	139	137	243	**81**	231
P9	3	L	81	**51**	189	225	99
s232	2	L	**817**	2,817	3,645	10,449	3,681
Average (overall)			**24,053**	50,809	156,837	678,973	251,294

In the experimental analysis an extra subdividing step was executed on iterations $k = \lambda^N$, where λ is equal either to 2 or 5, and $N = 1, 2, \ldots$.

Experimental investigation

Group 2: performance test on 67 constrained test problems

Number of function evaluations and time (in sec.) solving problems from *DIRECTLib*

Label	DIRECT- GLH		DIRECT- NAS		DIRECT- Barrier		DIRECT-sub-div ($\lambda = 2$)		DIRECT-sub-div ($\lambda = 5$)	
Average (overall)	**14.74**	**99, 472**	5, 220.33	237, 875	602.64	550, 537	413.34	653, 228	581.86	581.86
Average (NL cons.)	**9.66**	**96, 262**	6, 417.90	286, 286	969.32	551, 735	735.42	582, 621	967.62	590, 223
Average (L cons.)	**19.68**	**102, 587**	4, 057.97	190, 888	246.74	549, 374	100.73	721, 758	207.45	523, 720
Average ($n \geq 4$)	**31.67**	**218, 047**	11, 559.16	525, 766	252.57	757, 882	99.59	845, 201	379.33	757, 432
Average ($n \leq 3$)	**1.02**	**3, 330**	81.73	4, 450	886.48	382, 419	667.73	497, 573	746.07	393, 538
Median	**0.32**	**3, 425**	9.69	5, 099	91.98	$> 10^6$	54.27	$> 10^6$	83.95	$> 10^6$
# of failed	**5**		15		35		43		36	

In the experimental analysis an extra subdividing step was executed on iterations $k = \lambda^N$, where λ is equal either to 2 or 5, and $N = 1, 2, \ldots$.

Publikacijos

- **Priimta publikacija:**

  L. Stripinis, L. G. Casado, J. Žilinskas, R. Paulavičius (2021).
  On MATLAB experience in accelerating DIRECT-GLce algorithm for constrained global optimization through dynamic data structures and parallelization.
  Applied Mathematics and Computation. ISSN: 0096-3003. 2021, vol. 390, p. 1-17. DOI: 10.1016/j.amc.2020.125596;

- **Įteikta publikacija:**

  L. Stripinis, R. Paulavičius (2020).
  A modified DIRECT-GL algorithm for global optimization with hidden constraints.

  Optimization Letters; (Vyksta recenzavimas);

Dalyvavimas mokslo projektuose

Dalyvavimas mokslo projektuose:

- Lietuvos mokslo tarybos finansuojamame, "Dviejų lygmenų opmizavimo algoritmų kūrimas ir taikymai" (Nr. P-MIP-17-60).

4 mokslo metų suvestinė

Atsiskaityti egzaminai:

- Lygiagretieji ir paskirstytieji skaičiavimai, Įvertinimas: 9;
  - prof. dr. (HP) Julius ŽILINSKAS
- Optimizacijos teorija, algoritmų sudėtingumas, Įvertinimas: 8;
  - prof. habil. dr. Antanas ŽILINSKAS
- Globaliojo optimizavimo metodai, Įvertinimas: 7;
  - prof. habil. dr. Antanas ŽILINSKAS
- Informatikos matematiniai metodai, Įvertinimas: 9;
  - prof. dr. (HP) Julius ŽILINSKAS

4 mokslo metų suvestinė

Moksliniai rezultatai pristatyti tokiose konferencijose:

- **L. Stripinis**, R. Paulavičius, J. Žilinskas. Importance of optimization techniques for the social sciences, The International EURO mini Conference Modelling and Simulation of Social-Behavioural Phenomena in Creative Societies, 2019 September 18–20, Vilnius, Lithuania (Plenary Session);
- **L. Stripinis**, R. Paulavičius. Improved DIRECT-type Algorithms for Generally Constrained Global Optimization Problems, 10th International Workshop on DATA ANALYSIS METHODS FOR SOFTWARE SYSTEMS, 2018 November 29 – December 1, Druskininkai, Lithuania (Poster Session)
- **L. Stripinis**, J. Žilinskas, R. Paulavičius. Improved DIRECT-type algorithm for constrained global optimization problems, EUROPT 2018: 16th EUROPT Workshop on Advances in Continuous Optimization, 2018 July 12–13, Almeria, Spain (Plenary Session);
- **L. Stripinis**, R. Paulavičius. Improved DIRECT-type Algorithms for Generally Constrained Global Optimization Problems, 9th International Workshop on DATA ANALYSIS METHODS FOR SOFTWARE SYSTEMS, 2017 November 30 – December 2, Druskininkai, Lithuania (Poster Session);
- R. Paulavičius, L. Stripinis, **J. Žilinskas**. DIRECT-type algorithms for constrained global optimization, EUROPT 2017: 15th EUROPT Workshop on Advances in Continuous Optimization, 2017 July 12–14, Montreal, Canada (Plenary Session);

4 mokslo metų suvestinė

Atspausdintos publikacijos:

- L. Stripinis, R. Paulavičius, Žilinskas (2019).
  Penalty functions and two-step selection procedure based DIRECT-type algorithm for constrained global optimization.
  Structural and Multidisciplinary Optimization, ISSN 1615-1488, DOI: 10.1007/s00158-018-2181-2;
- L. Stripinis, R. Paulavičius, Žilinskas (2018).
  Improved scheme for selection of potentially optimal hyper-rectangles in DIRECT.
  Optimization Letters, ISSN 1862-4472, 12 (7), 1699-1712, DOI: 10.1007/s11590-017-1228-4;

Priimta publikacijos:

- L. Stripinis, L. G. Casado, J. Žilinskas, R. Paulavičius (2021).
  On MATLAB experience in accelerating DIRECT-GLce algorithm for constrained global optimization through dynamic data structures and parallelization.
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AČIŪ UŽ DĖMESĮ!