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SOLUTION OF A PSEUDOPARABOLIC EQUATION WITH NONLOCAL INTEGRAL CONDITIONS BY THE FINITE DIFFERENCE METHOD

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VILNIAUS UNIVERSITETAS

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PSEUDOPARABOLINĖS LYGTIES SU NELOKALIOSIOMIS INTEGRALINĖMIS SĄLYGOMIS SPRENDIMAS BAIGTINIŲ SKIRTUMŲ METODU

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Introduction

Statement of the problem

In this thesis difference schemes for the third-order pseudoparabolic equations with nonlocal integral conditions are considered.

Topicality of the problem

During the recent decades there have been extensive investigations of the pseudoparabolic equation

$$\frac{\partial u}{\partial t} - k\frac{\partial \Delta u}{\partial t} - \Delta u = f(x). \tag{1}$$

Many important results regarding the existence, uniqueness and other properties of the solution of this equation have been published.

Equation (1) has opened new possibilities of modelling various physical processes mathematically. For example, G. Barenblatt et al. [1], E. DiBenedetto and M. Pierre [12], B.D. Coleman et al. [11] used this equation as model of diffusion of fluid in fractured porous media. It can also serve as a model of heat conduction involving a thermodynamic temperature $T = u - k\Delta u$ and a conductive temperature $u(x,t)$: P.J. Chen and M.E. Gurtin [4].

In 1973, W.H. Ford and T.W. Ting wrote the article *"Stability and Convergence of Difference Approximations to Pseudoparabolic Partial Differential Equations"* [13]. It is one of the first works, where the finite difference method is investigated for a pseudoparabolic equation with classical boundary conditions. The existence and uniqueness of the solution as wel as the convergence of the finite difference method have been proved by investigating the spectrum properties of matrices of a system of difference problems.

In this dissertation, solution of a pseudoparabolic equation with nonlocal conditions is considered by the finite difference method.

Problems with nonlocal conditions are one of the parts of a rapidly developing theory of differential equations. Nonlocal conditions emerge when the values of the solution in quest or it its derivatives at boundary points are associated with the values at other boundary or internal points of the domain. In terms of practice, nonlocal conditions appear in case there is no possibility to measure data at the boundary of the domain of the problem considered. If there is a boundary point in nonlocal conditions, then conditions of this kind are called nonlocal boundary conditions.

In 1963, differential equations with nonlocal conditions were started to investigate by J.R. Cannon [3]. He investigated a parabolic problem where a nonlocal condition was formulated instead of the one boundary condition.

A bit later L.I. Kamynin considered a similar problem [17].

Parabolic equations with nonlocal integral conditions are also investigated by A. Bouziani [2, 19], N.I. Ionkin [15], A.V. Gulin [14], S. Mesloub [19, 18], N.I. Yurchuk [30], R. Čiegis [7, 8, 9], S. Pečiulytė [20], M. Sapagovas [23, 25, 26, 27], A. Štikonas [9, 20] and others.

A.F. Chudnovskij has formulated and analyzed a nonlocal pseudoparabolic problem in the monograph *"Teplofizika pochv"* [6]. A survey of problems on numerical modeling of moisture dynamics in soil is presented in it. For the first time this problem was formulated by the same author in 1969 [5].

Research object

The main research object here is third-order one-and two-dimensional linear differential pseudoparabolic equations with nonlocal conditions.

Aim and tasks of the work

The aim of this dissertation is to investigate the solution of the one-and two-dimensional pseudoparabolic equation with nonlocal conditions by difference methods, to explore the conditions of stability of the obtained difference schemes subject to the parameters in nonlocal conditions.

The main tasks of the work are:

1. To analyze the finite difference method for a one-dimensional pseudoparabolic equation with nonlocal integral conditions, to investigate the existence and uniqueness of the solution and the stability of a difference scheme.
2. To consider the solution of a two-dimensional pseudoparabolic equation with integral conditions applying the locally one-dimensional method.

3. To investigate the stability of locally one-dimensional difference schemes for a pseudoparabolic equation.

4. To form and analyze a difference scheme of higher accuracy for a one-dimensional pseudoparabolic equation with integral conditions.

5. To explore explicit difference schemes for a pseudoparabolic equation.

Methodology of research

In this work, the analytical method is applied in the investigation of solutions of difference schemes. The spectrum structure of difference operators is considered. The locally one-dimensional method is applied in solving a two-dimensional pseudoparabolic equation with nonlocal conditions. The numerical experiment and mathematical modeling method is also used in this thesis. Mathematical software packages: Mathcad, Maple and MathLab were used when doing numerical experiments.

Scientific novelty

Scientists usually consider pseudoparabolic equations with classical conditions. In this thesis, a pseudoparabolic equation with nonlocal conditions is considered. The finite difference method has been applied here and to investigate a one-dimensional pseudoparabolic equation with nonlocal conditions. Whereas, the finite difference method was investigated for a pseudoparabolic equation only with the classical conditions till now.

A two-dimensional pseudoparabolic equation and its solution has been considered applying the locally one-dimensional method. This method renders an opportunity to reduce a two-dimensional problem to one-dimensional problems and thus to simplify the solution of this problem.

Three-layer explicit difference schemes are also investigated for a one-dimensional pseudoparabolic equation with nonlocal conditions. This work extends and supplements the results of other scientists when considering difference schemes for pseudoparabolic problems.

Practical value of the results

The results obtained in the doctoral thesis might be applied in the consideration of the existence and uniqueness of the solution to differential and difference problems, when solving systems of difference equations by iterative methods and investigating the stability of difference schemes for pseudoparabolic equations. Mathematical models of a pseudoparabolic equation with nonlocal conditions are important in solving practical problems connected with the dynamics of ground water.

Defended propositions

- The methodology for spectrum analysis of a difference operator for a one-dimensional linear pseudoparabolic equation with nonlocal integral conditions of two types.

- Difference schemes for a one-and two-dimensional pseudoparabolic equation with the approximation error $O(\tau^2 + h^2)$, the stability conditions of these schemes.

- Locally one-dimensional schemes, the stability of these schemes for a two-dimensional pseudoparabolic equation with nonlocal integral conditions in one coordinate direction.

- Explicit difference schemes for a pseudoparabolic equation with classical and nonlocal conditions.

The scope of the scientific work

The doctoral thesis consists of the introduction, four chapters, conclusions, the list of references, and that of author's publications. The total scope of the doctoral dissertation is 89 pages, 7 figures and 7 tables. The results of the doctoral dissertation are presented in 5 publications. The results were also presented at 2 national and 5 international conferences. The language of the doctoral dissertation is Lithuanian.

Chapter 1. The finite difference method for a one-dimensional pseudoparabolic equation

We consider the third-order pseudoparabolic equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \eta \frac{\partial^3 u}{\partial t \partial x^2} + f(x,t),\ 0 < x < 1,\ t \geqslant 0 \tag{2}$$

subject to the initial condition

$$u(x,0) = \varphi(x), \tag{3}$$

and to the integral conditions

$$\int_0^1 u(x,t)\mathrm{d}x = \mu_1(t), \tag{4}$$

$$\int_0^1 x\,u(x,t)\mathrm{d}x = \mu_2(t), \tag{5}$$

where μ_1, μ_2 are given functions that are sufficiently smooth.

The aim is to investigate the stability of a difference scheme for problem (2)–(5).

We approximate differential problem (2)–(5) by the following system of difference equations:

$$\frac{u_i^{n+1} - u_i^n}{\tau} = \frac{u_{i-1}^{n+1} - 2u_i^{n+1} + u_{i+1}^{n+1}}{h^2} + \\ + \eta\Big(\frac{u_{i-1}^{n+1} - 2u_i^{n+1} + u_{i+1}^{n+1}}{h^2} - \frac{u_{i-1}^{n} - 2u_i^{n} + u_{i+1}^{n}}{h^2}\Big) \cdot \frac{1}{\tau} + f_i^{n+1}, \tag{6}$$

$$u_i^0 = \varphi_i, \tag{7}$$

$$l_1\Big(u^{n+1}\Big) \equiv h\Big(\frac{u_0^{n+1} + u_N^{n+1}}{2} + \sum_{i=1}^{N-1} u_i^{n+1}\Big) = \mu_1^{n+1}, \tag{8}$$

$$l_2\Big(u^{n+1}\Big) \equiv h\Big(\frac{u_N^{n+1}}{2} + \sum_{i=1}^{N-1} ihu_i^{n+1}\Big) = \mu_2^{n+1}, \tag{9}$$

$$j = 0, 1, \ldots, M-1,\ i = 1, \ldots, N-1,\ h = 1/N,\ \tau = \frac{T}{M}.$$

The system of difference equations (6)–(9) always has a unique solution found in the way

$$u_i^{n+1} = c_1(u_i^{n+1})^1 + c_2(u_i^{n+1})^2 + (u_i^{n+1})^0, \quad i = \overline{0, N}, \tag{10}$$

Here $(u_i^{n+1})^1, (u_i^{n+1})^2, (u_i^{n+1})^0$ are the solutions of the following problems:

$$\begin{cases} b(u_{i-1}^{n+1})^1 + a(u_i^{n+1})^1 + b(u_{i+1}^{n+1})^1 = 0, \quad i = \overline{1, N-1}, \\ (u_0^{n+1})^1 = 1, \ (u_N^{n+1})^1 = 0, \end{cases}$$

$$\begin{cases} b(u_{i-1}^{n+1})^2 + a(u_i^{n+1})^2 + b(u_{i+1}^{n+1})^2 = 0, \quad i = \overline{1, N-1}, \\ (u_0^{n+1})^2 = 0, \ (u_N^{n+1})^2 = 1, \end{cases}$$

$$\begin{cases} b(u_{i-1}^{n+1})^0 + a(u_i^{n+1})^0 + b(u_{i+1}^{n+1})^0 = f_i^{n+1}, \quad i = \overline{1, N-1}, \\ (u_0^{n+1})^0 = 0, \ (u_N^{n+1})^0 = 0. \end{cases}$$

We find the coefficients c_1, c_2 from the following system:

$$\begin{cases} l_1\Big((u^{n+1})^1\Big)c_1 + l_1\Big((u^{n+1})^2\Big)c_2 = F_0^n - l_1\Big((u^{n+1})^0\Big), \\ l_2\Big((u^{n+1})^1\Big)c_1 + l_2\Big((u^{n+1})^2\Big)c_2 = F_N^n - l_2\Big((u^{n+1})^0\Big). \end{cases}$$

Lemma 1 [see thesis, lemma 1.1]. *For all the values $\eta \geqslant 0, \tau > 0, h > 0$, system* (6)–(9) *has a unique solution* (10).

Let us investigate the stability of system (6)–(9). The main method for investigating the stability is analysis of the spectrum structure for the transition matrix of the system of difference equations.

We rewrite the difference scheme (6)–(9) on the $(n+1)$-layer in the following form

$$Bu^{n+1} = Cu^n + \tau f^{n+1}, \tag{11}$$

where

$$B = E + \tau\Lambda + \eta\Lambda, \quad C = E + \eta\Lambda \tag{12}$$

and $u_i^{n+1} = (u_1^{n+1}, \ u_2^{n+1}, \ ..., u_{N-1}^{n+1})^T$, B and C are matrices of order $(N-1)$,

$$\Lambda = h^{-2}\begin{pmatrix} 4-2h & 1-4h & 2-6h & \dots & 4h & 2h \\ -1 & 2 & -1 & \dots & 0 & 0 \\ 0 & -1 & 2 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 2 & -1 \\ 2h & 4h & 6h & \dots & 1-4h & 4-2h \end{pmatrix}.$$

If there exists B^{-1}, we can express the difference scheme as follows

$$u^{n+1} = Su^n + \tau B^{-1} f^{n+1}. \tag{13}$$

The nonsymmetric matrix S is a transition matrix and it is defined by the formula

$$S = B^{-1}C.$$

Let us define the norm of arbitrary matrix M

$$\|M\|_* = \|H^{-1}MH\|_2, \quad \|u\|_* = \|H^{-1}u\|_2.$$

The matrix H is a matrix with columns that are linearly independent eigenvectors of the matrix Λ. In that case $\|S\|_* = \rho(S)$.

Then, we can use the condition of the stability of difference scheme (13)

$$|\lambda(S)| < 1. \tag{14}$$

Whereas, matrices B, C and Λ from expression (12) have the common system of eigenvectors, so we find the eigenvalues of matrix S by the formula

$$\lambda(S) = \frac{1 + \eta\lambda_k(\Lambda)}{1 + (\tau + \eta)\lambda_k(\Lambda)}.$$

Further, applying the analysis of the spectrum of the nonsymetric transition matrix, we investigate the stability of a difference scheme. Using the same technique of analysis of the structure for the spectrum as in articles [16, 25, 28], we prove that all the eigenvalues of the transition matrix S are real and positive.

We have to find all the eigenvalues of matrix Λ, i.e., all the eigenvalues of the difference operator

$$\frac{u_{i-1} - 2u_i + u_{i+1}}{h^2} + \lambda u_i = 0, \quad i = 1, \dots, N-1, \tag{15}$$

$$h\Big(\frac{u_0 + u_N}{2} + \sum_{i=1}^{N-1} u_i\Big) = 0, \tag{16}$$

$$h\Big(\frac{u_N}{2} + \sum_{i=1}^{N-1} ih\, u_i\Big) = 0. \tag{17}$$

The following statements have been proved.

Theorem 1 [see thesis, theorem 1.1]. *All the positive eigenvalues of the difference operator* (15)–(17) *satisfying the inequality* $|1 - \lambda h^2/2| \leqslant 1$, *are of the form*

$$\lambda_k = \frac{4}{h^2} \sin^2 \frac{\alpha_k h}{2}, \tag{18}$$

where α_k are roots either of the equation

$$\sin \frac{\alpha}{2} = 0 \tag{19}$$

or the equation

$$\tan \frac{\alpha}{2} = \frac{N}{2} \sin \alpha h. \tag{20}$$

Theorem 2 [see thesis, theorem 1.2]. *All the eigenvalues of difference problem* (15)–(17) *are real and positive.*

All the eigenvalues of eigenvalue problem (15)–(17) are distinct. It means that the matrix Λ (as well as matrix S) is a simple-structured matrix.

Theorem 3 [see thesis, theorem 1.3]. *Difference scheme* (6)–(9) *is stable with all the values of parameters* $\eta > 0$, $\tau > 0$, $h > 0$.

Next, we consider the third-order pseudoparabolic equation (2) with initial condition (3), and nonlocal integral conditions

$$u(0,t) = \gamma_1 \int_0^1 u(x,t)\mathrm{d}x + \mu_1(t), \tag{21}$$

$$u(1,t) = \gamma_2 \int_0^1 u(x,t)\mathrm{d}x + \mu_2(t). \tag{22}$$

instead of nonlocal conditions (4)–(5).

We approximate differential problem (2), (3), (21), (22) by the following system of difference equations

$$\frac{u_i^{n+1} - u_i^n}{\tau} = \frac{u_{i-1}^{n+1} - 2u_i^{n+1} + u_{i+1}^{n+1}}{h^2} + \\ + \eta\left(\frac{u_{i-1}^{n+1} - 2u_i^{n+1} + u_{i+1}^{n+1}}{h^2} - \frac{u_{i-1}^{n} - 2u_i^{n} + u_{i+1}^{n}}{h^2}\right)\frac{1}{\tau} + f_i^n, \tag{23}$$

$$u_i^0 = \varphi_i, \tag{24}$$

$$u_0^{n+1} = \gamma_1 h\biggl(\frac{u_0^{n+1}+u_N^{n+1}}{2} + \sum_{i=1}^{N-1} u_i^{n+1}\biggr) + \mu_1^{n+1}, \quad j=\overline{1, M-1}, \tag{25}$$

$$u_N^{n+1} = \gamma_2 h\biggl(\frac{u_0^{n+1}+u_N^{n+1}}{2} + \sum_{i=1}^{N-1} u_i^{n+1}\biggr) + \mu_2^{n+1}, \quad j=\overline{1, M-1}, \tag{26}$$

To establish the stability of a difference scheme, we need to investigate the spectral structure of the difference operator Λ with nonlocal conditions. In other words, we will find all the eigenvalues of matrix Λ, i.e., all the eigenvalues of the difference operator

$$\frac{u_{i-1}-2u_i+u_{i+1}}{h^2} + \lambda u_i = 0, \quad i=\overline{1, N-1}, \tag{27}$$

$$u_0 = \gamma_1 h\biggl(\frac{u_0+u_N}{2} + \sum_{i=1}^{N-1} u_i\biggr), \tag{28}$$

$$u_N = \gamma_2 h\biggl(\frac{u_0+u_N}{2} + \sum_{i=1}^{N-1} u_i\biggr). \tag{29}$$

To this end, we apply the same technique as in [10, 22, 23, 29].

If $\gamma_1+\gamma_2 = 2$, then there exists an eigenvalue $\lambda = 0$. Difference problem (27)–(29) has only one negative eigenvalue, if $\gamma_1+\gamma_2 > 2$.

When $\gamma_1+\gamma_2 < 2$, in all the three cases ($\gamma_1+\gamma_2 < 0$, $0 < \gamma_1+\gamma_2 < 2$, $\gamma_1+\gamma_2 = 0$), we have $N-1$ real eigenvalues. It follows that there are no complex eigenvalues.

Let us consider when this inequality $|\lambda_k(S)| < 1$ is true.

Recall the expression of the eigenvalue of matrix S

$$\lambda_k(S) = \frac{1+\eta\lambda_k(\Lambda)}{1+(\tau+\eta)\lambda_k(\Lambda)}.$$

If $\gamma_1+\gamma_2 < 2$, then the eigenvalues $\lambda_k(\Lambda)$ of matrix Λ are positive.

The sufficient condition of stability. The difference scheme (23)–(26) is stable if

$$|\lambda_k(S)| < 1,$$

i.e., if $\gamma_1+\gamma_2 < 2$ and $\eta \geqslant 0$.

If $\gamma_1+\gamma_2 > 2$, then there exists one negative eigenvalue $\lambda_k(\Lambda)$ and $|\lambda_k(S)| > 1$. The difference scheme is not stable.

Chapter 2. Difference schemes of increased order for the approximation to one-and two-dimensions

In this chapter, we consider an implicit difference scheme for pseudoparabolic equation (2) with integral conditions (21)–(22). Let us write a difference equation for pseudoparabolic equation (2) in such a form

$$\frac{u_i^{n+1} - u_i^n}{\tau} = \sigma \Lambda u_i^{n+1} + (1-\sigma)\Lambda u_i^n + \varphi_i^{n+1}, \tag{30}$$

$$\Lambda u_i^n = \frac{u_{i-1}^n - 2u_i^n + u_{i+1}^n}{h^2}, \quad i = \overline{1, N-1}.$$

If the parameter σ is well chosen ($\sigma = 1/2 + \eta/\tau$), then the difference scheme is with the approximation error $O(h^2 + \tau^2)$.

So we replace equation (2) with conditions (21)–(22) by the following difference scheme:

$$\frac{u_i^{n+1} - u_i^n}{\tau} = \sigma \Lambda u_i^{n+1} + (1-\sigma)\Lambda u_i^n + \varphi_i^{n+1}, \tag{31}$$

$$\tilde{u}_0^{n+1} = \gamma_1 h \left(\frac{\tilde{u}_0^{n+1} + \tilde{u}_N^{n+1}}{2} + \sum_{i=1}^{N-1} \tilde{u}_i^{n+1} \right) + \tilde{\mu}_1^{n+1}, \tag{32}$$

$$\tilde{u}_N^{n+1} = \gamma_2 h \left(\frac{\tilde{u}_0^{n+1} + \tilde{u}_N^{n+1}}{2} + \sum_{i=1}^{N-1} \tilde{u}_i^{n+1} \right) + \tilde{\mu}_2^{n+1}. \tag{33}$$

where

$$\tilde{u}_i^{n+1} = \sigma u_i^{n+1} + (1-\sigma) u_i^n,$$

Lemma 3 [see thesis, lemma 2.1]. *If the solution $u(x,t)$ is smooth enough and*

$$\sigma = \frac{1}{2} + \frac{\eta}{\tau} \quad \text{ir} \quad \varphi_i^{n+1} = f_i^{n+1/2},$$

difference equation (31) *approximates differential equation* (2) *with the truncation error* $O(h^2 + \tau^2)$.

Further we consider the two-dimensional pseudoparabolic equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \eta \frac{\partial}{\partial t}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + f(x,y,t), \tag{34}$$

with nonlocal conditions

$$u(0,y,t)=\gamma_1\int_0^1 u(x,y,t)\mathrm{d}x+\mu_1(y,t), \tag{35}$$

$$u(1,y,t)=\gamma_2\int_0^1 u(x,y,t)\mathrm{d}x+\mu_2(y,t), \tag{36}$$

$$u(x,0,t)=\mu_3(x,t),\quad u(x,1,t)=\mu_4(x,t), \tag{37}$$

$$u(x,y,0)=\varphi(x,y), \tag{38}$$

Now we replace equation (34) by the following difference equation

$$\frac{u_{ij}^{n+1}-u_{ij}^n}{\tau}=\sigma(\Lambda_1+\Lambda_2)u_{ij}^{n+1}+(1-\sigma)(\Lambda_1+\Lambda_2)u_{ij}^n+\varphi_{ij}^{n+1}, \tag{39}$$

where $\sigma=1/2+\eta/\tau$, $\varphi_{ij}^{n+1}=f_{ij}^{n+1/2}$, and

$$\Lambda_1 u_{ij}^n=\frac{u_{i-1,j}^n-2u_{ij}^n+u_{i+1,j}^n}{h^2},\quad \Lambda_2 u_{ij}^n=\frac{u_{i,j-1}^n-2u_{ij}^n+u_{i,j+1}^n}{h^2},\quad i=\overline{1,N-1}.$$

Using the proof of Lemma 3, we can prove that difference equation (39) approximates differential equation (34) with the truncation error $O(h^2+\tau^2)$.

Chapter 3. The locally one-dimensional method for solving of two-dimensional pseudoparabolic equation with integral conditions

In this chapter, we study a two-dimensional linear pseudoparabolic equation with nonlocal integral boundary conditions in one coordinate direction. The locally one-dimensional method is used for solving this problem. We have proved the stability of a finite difference scheme, based on the structure of spectrum of the difference operator with nonlocal conditions.

We consider the third-order two-dimensional pseudoparabolic problem (34)–(38) in the domain $\Omega_T=\{0<x,y<1,\ 0<t<T\}$.

We propose a locally one-dimensional scheme for problem (34)–(38). The idea of decomposition of a parabolic problem into a chain of one-dimensional differential problems was considered in [21]. Following this idea, we construct a chain of one-dimensional pseudoparabolic problems

$$\frac{1}{2}\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}+\eta\frac{\partial}{\partial t}\Big(\frac{\partial^2 u}{\partial x^2}\Big)+\frac{1}{2}f, \tag{40}$$

$$\frac{1}{2}\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial y^2}+\eta\frac{\partial}{\partial t}\Big(\frac{\partial^2 u}{\partial y^2}\Big)+\frac{1}{2}f. \tag{41}$$

Equation (41) is approximated in the interval $(t_n,\ t_{n+1/2})$ and equation (40) – in the interval $(t_{n+1/2},\ t_{n+1})$.

In order to go from layer $t = t_n$ to layer $t = t_{n+1}$, we replace equations (40), (41) with conditions (35)–(37) by the following one-dimensional difference scheme:

$$\frac{u_{ij}^{n+1/2} - u_{ij}^n}{\tau} = \sigma \Lambda_1 u_{ij}^{n+1/2} + (1-\sigma)\Lambda_1 u_{ij}^n + \varphi_{ij}^{n+1/2}, \tag{42}$$

$$\tilde{u}_{0j}^{n+1/2} = \gamma_1 h \left(\frac{\tilde{u}_{0j}^{n+1/2} + \tilde{u}_{Nj}^{n+1/2}}{2} + \sum_{i=1}^{N-1} \tilde{u}_{ij}^{n+1/2} \right) + \tilde{\mu}_{1j}^{\ n+1/2}, \tag{43}$$

$$\tilde{u}_{Nj}^{n+1/2} = \gamma_2 h \left(\frac{\tilde{u}_{0j}^{n+1/2} + \tilde{u}_{Nj}^{n+1/2}}{2} + \sum_{i=1}^{N-1} \tilde{u}_{ij}^{n+1/2} \right) + \tilde{\mu}_{2j}^{\ n+1/2}, \tag{44}$$

$$\frac{u_{ij}^{n+1} - u_{ij}^{n+1/2}}{\tau} = \sigma \Lambda_2 u_{ij}^{n+1} + (1-\sigma)\Lambda_2 u_{ij}^{n+1/2} + \varphi_{ij}^{n+1}, \tag{45}$$

$$u_{i0}^{n+1} = \mu_{3i}, \quad u_{iN}^{n+1} = \mu_{4i}, \tag{46}$$

where

$$\Lambda_1 u_{ij}^{n+1} = \frac{u_{i-1,j}^{n+1} - 2u_{ij}^{n+1} + u_{i+1,j}^{n+1}}{h^2}, \qquad \Lambda_2 u_{ij}^{n+1} = \frac{u_{i,j-1}^{n+1} - 2u_{ij}^{n+1} + u_{i,j+1}^{n+1}}{h^2}$$

$$\tilde{u}_{ij}^{n+1/2} = \sigma u_{ij}^{n+1/2} + (1-\sigma) u_{ij}^n,$$

$$\tilde{\mu}_{kj}^{\ n+1/2} = \sigma \mu_{kj}^{\ n+1/2} + (1-\sigma)\mu_{kj}^{\ n+1/2}, \quad k = 1, 2.$$

Now we write difference scheme (42)–(46) in the usual matrix form:

$$u^{n+1} = S u^n + b^n, \tag{47}$$

where u^n and b^n are $(N-1)^2$-dimensional vectors, S is a matrix of order $(N-1)^2$, and the expression of matrix S is

$$S = (I + \tau\sigma A_2)^{-1}(I - \tau(1-\sigma)A_2)(I + \tau\sigma A_1)^{-1}(I - \tau(1-\sigma)A_1). \tag{48}$$

Matrices A_1 and A_2 have the same system of linearly independent eigenvectors. Hence it follows [24] that

$$A_1 A_2 = A_2 A_1.$$

Theorem 4 [see thesis, theorem 3.1]. *The eigenvalues of matrix S are*

$$\lambda(S) = q(\lambda(\Lambda_x)) \cdot q(\lambda(\Lambda_y)), \tag{49}$$

where

$$q(\lambda) = \frac{1 - \tau(1-\sigma)\lambda}{1 + \tau\sigma\lambda}. \tag{50}$$

Theorem 5 [see thesis, theorem 3.2]. *If $\gamma_1 + \gamma_2 < 2$, then the difference scheme* (42)–(46) *is stable for all values h and τ.*

Chapter 4. Three-layer explicit difference schemes for a one-dimensional pseudoparabolic equation

The main purpose of this chapter is to investigate the possibilities of constructing three-layer explicit difference schemes.

First of all, let us take a differential parabolic equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + f(x,t), \tag{51}$$

and apply the difference scheme of Dufort-Frankel to this equation

$$\frac{u_i^{n+1} - u_i^{n-1}}{2\tau} = \frac{u_{i-1}^n - (u_i^{n+1} + u_i^{n-1}) + u_{i+1}^n}{h^2} + f_i^n. \tag{52}$$

We add an additional term $\eta\frac{\partial}{\partial t}(\frac{\partial^2 u}{\partial x^2})$ to equation (51), and we get a pseudoparabolic equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \eta\frac{\partial}{\partial t}\left(\frac{\partial^2 u}{\partial x^2}\right) + f(x,t). \tag{53}$$

In some cases, as $\eta > 0$ is a small value or as $\eta \to 0$, the term $\eta\frac{\partial}{\partial t}(\frac{\partial^2 u}{\partial x^2})$ could be interpreted as a regularizator of parabolic equation (51). In other words, with this third-order term it is possible to secure a special desirable feature of a numerical method or mathematic model. In this chapter, we use is idea for constructing explicit difference schemes for a pseudoparabolic equation.

In order to write explicit difference schemes, it means using different approximations of the regularizator $\eta\frac{\partial}{\partial t}(\frac{\partial^2 u}{\partial x^2})$, usually we get nonstable difference schemes.

Thus, let us take a term of regularization and use such an approximation

$$\left(\frac{\partial}{\partial t}\left(\frac{\partial^2 u}{\partial x^2}\right)\right)_i^n = \left(\frac{\partial^2}{\partial x^2}\left(\frac{\partial u}{\partial t}\right)\right)_i^n =$$
$$= \frac{1}{\tau}\left(\frac{u_{i-1}^n - 2u_i^{n+1} + u_{i+1}^n}{h^2} - \frac{u_{i-1}^{n-1} - 2u_i^n + u_{i+1}^{n-1}}{h^2}\right) + O(\frac{\tau}{h^2} + h^2)$$

that difference scheme (52) would be explicit. So we write the following difference scheme for pseudoparabolic equation (53)

$$\frac{u_i^{n+1} - u_i^n}{\tau} = \tau \Lambda u_i^n + \eta \frac{u_{i-1}^n - 2u_i^{n+1} + u_{i+1}^n}{h^2} - \eta \frac{u_{i-1}^{n-1} - 2u_i^n + u_{i+1}^{n-1}}{h^2} + f_i^n, \tag{54}$$

with the conditions

$$u_0^n = \mu_1^n, \quad u_N^n = \mu_2^n, \quad u_i^0 = \varphi_i. \tag{55}$$

Analyzing the stability of explicit difference schemes, we write equation (54) with boundary conditions (55) in the form as follows

$$Eu^{n+1} = (E - (\tau + \eta)\Lambda)u^n + \eta \Lambda u^{n-1} + \tau \overline{f}^n, \tag{56}$$

$\overline{f}^n$-is the known vector and it is composed of f_i^n and $\mu_1, \ \mu_2$.

We write equation (56) in a more general form

$$Au^{n+1} + Bu^n + Cu^{n-1} = f^n. \tag{57}$$

After joining the identity $u^n \equiv u^n$ to equation (57) and after writing this equation in the expression below

$$u^{n+1} = -A^{-1}Bu^n - A^{-1}Cu^{n-1} + A^{-1}f^n, \tag{58}$$

we get

$$\begin{pmatrix} u^{n+1} \\ u^n \end{pmatrix} = \begin{pmatrix} -A^{-1}B & -A^{-1}C \\ E & 0 \end{pmatrix} \begin{pmatrix} u^{n+1} \\ u^n \end{pmatrix} + \begin{pmatrix} A^{-1}f^n \\ 0 \end{pmatrix}. \tag{59}$$

Let us define vectors z^{n+1} and $\tilde{f}^n$ of order $2(N-1)$

$$z^{n+1} = \begin{pmatrix} u^{n+1} \\ u^n \end{pmatrix}, \quad \tilde{f}^n = \begin{pmatrix} A^{-1}f^n \\ 0 \end{pmatrix},$$

and matrix S of order $2(N-1)$:

$$S = \begin{pmatrix} -A^{-1}B & -A^{-1}C \\ E & 0 \end{pmatrix}.$$

Then equation (58), i.e. a three-layer difference scheme (54)–(55), can be written in the following form

$$z^{n+1} = Sz^n + \tilde{f}^n. \tag{60}$$

Afterwards, we solve a nonlinear eigenvalue problem

$$\mu^2 Av + \mu Bv + Cv = 0 \tag{61}$$

and we find the eigenvalue μ.

In order to get a stable difference scheme it is necessary, that

$$|\mu| < 1.$$

We prove that $|\mu| < 1$, if $\lambda > 0$ and $\tau \leqslant h^2/4$. Hence we conclude that difference scheme (54) is conditionally stable. The numerical results also corroborate that difference scheme (54) is stable.

Similarly we have done numerical experiments for such explicit difference schemes

$$\frac{u_i^{n+1} - u_i^{n-1}}{2\tau} = \frac{u_{i-1}^n - (u_i^{n+1} + u_i^{n-1}) + u_{i+1}^n}{h^2} + \\ + \frac{\eta}{\tau}\left(\frac{u_{i-1}^n - 2u_i^{n+1} + u_{i+1}^n}{h^2}\right) - \frac{\eta}{\tau}\left(\frac{u_{i-1}^{n-1} - 2u_i^n + u_{i+1}^{n-1}}{h^2}\right) + f_i^n,$$

$$\frac{u_i^{n+1} - u_i^n}{\tau} = \\ = \frac{u_{i-1}^n - 2u_i^n + u_{i+1}^n}{h^2} + \frac{\eta}{h^2}\left(\frac{u_{i-1}^{n+1} - u_{i-1}^n}{\tau} - 2\frac{u_i^{n+1} - u_i^n}{\tau} + \frac{u_{i+1}^n - u_{i+1}^{n-1}}{\tau}\right) + f_i^n.$$

We have obtained the results which show that the method is efficient.

General conclusions

1. The analysis of the transition matrix spectrum structure of difference schemes of a pseudoparabolic equation is an efficient method.

2. The locally one-dimensional method might be successfully applied to a two-dimensional pseudoparabolic equation with nonlocal conditions.

3. The well-known composition methods for explicit difference schemes of parabolic equations are not suitable for pseudoparabolic equations in general. The explicit difference schemes for pseudoparabolic equations composed by another principle are stable.

List of Published Works on the Topic of the Dissertation

In the reviewed scientific periodical publications

[A1] J. Jachimavičienė. Explicit difference schemes for pseudoparabolic equation with integral condition. *Liet. mat. rink. LMD darbai*, **53**:36–41, 2012.

[A2] J. Jachimavičienė and M. Sapagovas. Locally one-dimensional difference scheme for a pseudoparabolic equation with nonlocal conditions. *Lith. Math. J.*, **52**(1):53–61, 2012.

[A3] J. Jachimavičienė, Ž. Jesevičiūtė and M. Sapagovas. The stability of finite-difference schemes for a pseudoparabolic equation with nonlocal conditions. *Numer. Funct. Anal. Optimiz.*, **30**(9):988–1001, 2009.

In the other editions

[B1] J. Jachimavičienė. Solution of a third-order pseudoparabolic equation with nonlocal integral conditions by a finite-difference method. *MII Preprintas*, **2008-39**, 2008.

[B2] J. Jachimavičienė. The finite-difference method for a third-order pseudoparabolic equation with integral conditions. *Differ. Equ. and Their Appl. (DETA'2009), Proc. Intern. Conf.*, pp. 49–58, 2009.

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PSEUDOPARABOLINĖS LYGTIES SU NELOKALIOSIOMIS INTEGRALINĖMIS SĄLYGOMIS SPRENDIMAS BAIGTINIŲ SKIRTUMŲ METODU

Problemos formulavimas

Disertacijoje tiriamos skirtuminės schemos trečiosios eilės pseudoparabolinėms lygtims su nelokaliosiomis integralinėmis sąlygomis.

Darbo aktualumas

Per paskutinius dešimtmečius plačiai nagrinėta pseudoparabolinė lygtis

$$\frac{\partial u}{\partial t} - k\frac{\partial \Delta u}{\partial t} - \Delta u = f(x). \tag{0.1}$$

Išspausdinta daug svarbių rezultatų, susijusių su šios lygties sprendinio egzistavimu, vienatimi ir kitomis sprendinio savybėmis.

(0.1) lygtis atvėrė naujas galimybes matematiškai modeliuojant įvairius fizikinius procesus. G.Barenblatt, E.DiBenedetto, B.D.Coleman šią lygtį naudojo kaip skysčių sklidimo porėtose terpėse modelį. Tokią pat lygtį, kaip dviejų temperatūrų šilumos laidumo modelį, 1968 m. išnagrinėjo P.J.Chen ir M.E.Gurtin, kur $T = u - k\Delta u$ – termodinaminė temperatūra, o $u(x,t)$ – šilumos laidumo temperatūra.

1973 m. W.H.Ford ir T.W.Ting straipsnis *"Stability and Convergence of Difference Approximations to Pseudoparabolic Partial Differential Equations"* – vienas pirmųjų, kuriame teoriškai trečiosios eilės pseudoparabolinei lygčiai išnagrinėtas baigtinių skirtumų metodas. Skirtuminio uždavinio sprendinio egzistavimas ir vienatis, bei metodo konvergavimas įrodyti, nagrinėjant skirtuminių lygčių sistemos matricų spektro savybes.

Disertacijoje nagrinėjamas pseudoparabolinės lygties su nelokaliosiomis sąlygomis sprendimas baigtinių skirtumų metodu.

Uždaviniai su nelokaliosiomis sąlygomis yra viena iš sparčiai besivystančios diferencialinių lygčių teorijos dalių. Nelokaliosios sąlygos atsiranda tada, kai ieškomo sprendinio ar jo išvestinės reikšmės kraštiniuose taškuose yra susijusios su reikšmėmis kituose srities kraštiniuose ar vidiniuose taškuose. Praktiniu požiūriu, nelokaliosios sąlygos atsiranda tada, kai negalima tiesiogiai išmatuoti duomenų, nagrinėjamo uždavinio srities krašte. Kai nelokaliosiose sąlygose yra kraštinis taškas, tai tokias sąlygas vadiname nelokaliosiomis kraštinėmis sąlygomis.

Diferencialines lygtis su nelokaliosiomis sąlygomis 1963 m. vienas pirmųjų pradėjo nagrinėti J.R.Cannon. Jis nagrinėjo parabolinį uždavinį, kuriame nelokalioji integralinė sąlyga yra formuluojama vietoje vienos kraštinės sąlygos.

Panašų uždavinį kiek vėliau tyrė ir L.I.Kamyninas.

Parabolines lygtis su nelokaliosiomis integralinėmis sąlygomis taip pat nagrinėja A.Bouziani, A.V. Gulin, N.I.Ionkin, S.Mesloub, N.I.Yurchuk, R.Čiegis, S. Pečiulytė, M.Sapagovas, A.Štikonas ir kiti autoriai.

A.F. Čudnovskij monografijoje *"Teplofizika pochv"* suformuluotas ir išnagrinėtas nelokalusis pseudoparabolinis uždavinys. Čia pateikiama skaitinio modeliavimo dirvožemio drėgmės dinamikos uždaviniuose apžvalga. Toks uždavinys pirmą kartą buvo suformuluotas 1969 m. to paties autoriaus.

Tyrimų objektas

Disertacijos tyrimo objektas – trečiosios eilės vienmatės ir dvimatės tiesinės diferencialinės pseudoparabolinės lygtys su nelokaliosiomis sąlygomis.

Darbo tikslas ir uždaviniai

Disertacijos tikslas – išnagrinėti vienmatės ir dvimatės pseudoparabolinės lygties su nelokaliosiomis sąlygomis sprendimą skirtuminiais metodais, ištirti gautų skirtuminių schemų stabilumo sąlygas, priklausomai nuo parametrų nelokaliosiose sąlygose.

Siekiant numatyto tikslo buvo sprendžiami šie uždaviniai:

- išnagrinėti baigtinių skirtumų metodą vienmatei pseudoparabolinei lygčiai su nelokaliosiomis integralinėmis sąlygomis, ištirti apytikslio sprendinio egzistavimą, vienatį ir skirtuminės schemos stabilumą;
- išnagrinėti dvimatės pseudoparabolinės lygties su integralinėmis sąlygomis sprendimą lokaliai vienmačiu metodu;
- ištirti lokaliai vienmačių skirtuminių schemų pseudoparabolinei lygčiai stabilumą;
- sudaryti ir išnagrinėti padidinto tikslumo skirtuminę schemą vienmatei pseudoparabolinei lygčiai su integralinėmis sąlygomis;
- ištirti išreikštines skirtumines schemas pseudoparabolinei lygčiai.

Tyrimų metodika

Darbe taikomas analizinis skirtuminių lygčių sprendinių tyrimo metodas, nagrinėjama skirtuminių operatorių spektro struktūra. Sprendžiant dvimatę pseudoparabolinę lygtį su nelokaliosiomis sąlygomis pritaikytas lokaliai vienmatis metodas. Taip pat taikomi skaitinio eksperimento ir matematinio modeliavimo metodai. Atliekant skaitinius eksperimentus buvo naudojami Mathcad, Maple ir MathLab programų paketai.

Darbo mokslinis naujumas ir jo reikšmė

Daugelyje darbų mokslininkai nagrinėja pseudoparabolines lygtis su klasikinėmis sąlygomis. Šioje disertacijoje išnagrinėta pseudoparabolinė lygtis su nelokaliosiomis sąlygomis, pritaikytas ir išnagrinėtas baigtinių skirtumų metodas vienmatei pseudoparabolinei lygčiai su nelokaliosiomis sąlygomis. Tuo tarpu iki šiol baigtinių skirtumų metodas pseudoparabolinei lygčiai buvo išnagrinėtas tik su klasikinėmis sąlygomis.

Išnagrinėta dvimatė pseudoparabolinė lygtis ir jos sprendimas taikant lokaliai vienmatį metodą. Šis metodas suteikia galimybę dvimatį uždavinį suvesti į vienmačius ir taip supaprastinti uždavinio sprendimą.

Taip pat išnagrinėtos pseudoparabolinės lygties su nelokaliosiomis sąlygomis trisluoksnės išreikštinės skirtuminės schemos. Šie rezultatai praplečia ir papildo iki šiol kitų mokslininkų gautus rezultatus, nagrinėjant skirtumines schemas pseudoparaboliniams uždaviniams.

Darbo rezultatų praktinė reikšmė

Disertacijoje gauti rezultatai gali būti panaudojami nagrinėjant diferencialinio ir skirtuminio uždavinių sprendinio egzistavimą ir vienatį, skirtuminių lygčių sistemų sprendimui iteraciniais metodais bei skirtuminių schemų pseudoparabolinėms lygtims stabilumui tirti.

Pseudoparabolinės lygties su nelokaliosiomis sąlygomis matematiniai modeliai gali būti taikomi sprendžiant praktinius uždavinius, susijusius su gruntinių vandenų dinamika.

Ginamieji teiginiai

- Vienmatės tiesinės pseudoparabolinės lygties su dviejų tipų nelokaliosiomis sąlygomis skirtuminių schemų stabilumo nagrinėjimo būdas: skirtuminio operatoriaus spektro tyrimo metodika.
- Vienmatės ir dvimatės pseudoparabolinės lygties skirtuminės schemos su aproksimavimo paklaida $O(\tau^2 + h^2)$, šių schemų stabilumo sąlygos.
- Lokaliai vienmatės schemos, jų stabilumas dvimatei pseudoparabolinei lygčiai su nelokaliosiomis integralinėmis sąlygomis viena koordinačių kryptimi.

- Išreikštinės skirtuminės schemos pseudoparabolinei lygčiai su klasikinėmis ir nelokaliosiomis sąlygomis.

Disertacijos struktūra

Disertaciją sudaro įvadas, 4 skyriai, išvados, literatūros sąrašas ir autorės publikacijų disertacijos tema sąrašas. Bendra disertacijos apimtis – 89 puslapiai, 7 grafikai, 7 lentelės. Disertacijos rezultatai paskelbti 5 publikacijose.

Šia tema skaityta 7 pranešimai mokslinėse konferencijose.

Šios disertacijos *pirmajame skyriuje* nagrinėjama trečiosios eilės vienmatė pseudoparabolinė lygtis su skirtingomis dviejų tipų nelokaliosiomis integralinėmis sąlygomis. Nagrinėjamas baigtinių skirtumų metodas, formuluojamos sprendinio vienaties ir egzistavimo sąlygos. Užrašytos skirtuminės schemos ir taikant skirtuminio operatoriaus spektro struktūros tyrimą išnagrinėtas šių skirtuminių schemų stabilumas specialioje normoje.

Antrajame skyriuje nagrinėjama trečios eilės vienmatė ir dvimatė pseudoparabolinė lygtis su integralinėmis sąlygomis. Šio skyriaus pagrindinis tikslas – sudaryti ir išnagrinėti padidinto tikslumo skirtumines schemas žingsnio τ atžvilgiu. Įrodoma, kad skirtuminė lygtis aproksimuoja diferencialinę lygtį su paklaida $O(h^2 + \tau^2)$ bet kokioms τ ir h reikšmėms.

Trečiajame skyriuje skyriuje pateikiamas dvimačio uždavinio sprendimo algoritmas, taikant lokaliai vienmatį metodą. Gauti rezultatai panaudoti sprendinio egzistavimo ir vienaties sąlygoms formuluoti. Taip pat nagrinėjamas skirtuminių schemų stabilumo, priklausomai nuo parametrų $\gamma_1,\ \gamma_2$ klausimas.

Ketvirtajame disertacijos skyriuje nagrinėjamos trisluoknės išreikštinės skirtuminės schemos vienmatei pseudoparabolinei lygčiai tiek su klasikinėmis, tiek su nelokaliosiomis sąlygomis. Tiek teoriškai, tiek taikant skaitinius eksperimentus išnagrinėtas tokių skirtuminių schemų stabilumas.

Bendrosios išvados

1. Skirtuminių schemų pseudoparabolinei lygčiai su nelokaliosiomis integralinėmis sąlygomis perėjimo matricos spektro struktūros tyrimas yra efektyvus skirtuminių schemų stabilumo tyrimo metodas.

2. Dvimatėms tiesinėms pseudoparabolinėms lygtims su nelokaliosiomis sąlygomis spręsti sėkmingai galima taikyti lokaliai vienmatį metodą.

3. Parabolinio tipo lygtims žinomi išreikštinių skirtuminių schemų sudarymo metodai paprastai netinka pseudoparabolinėms lygtims. Išreikštinės skirtuminės schemos pseudoparabolinėms lygtims, sudarytos kitu principu, yra stabilios.

Trumpos žinios apie autorę

Justina Jachimavičienė gimė 1983 m. rugsėjo 24 d. Panevėžyje. 2006 m. įgijo matematikos bakalauro laipsnį Vytauto Didžiojo universitete Informatikos fakultete. 2008 m. baigė Taikomosios matematikos magistrantūros studijų programą ir įgijo matematikos magistro kvalifikacinį laipsnį Vytauto Didžiojo universitete Informatikos fakultete. 2008–2012 m. studijavo matematikos krypties doktorantūroje Vilniaus universiteto Matematikos ir informatikos institute.

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PSEUDOPARABOLINĖS LYGTIES
SU NELOKALIOSIOMIS INTEGRALINĖMIS SĄLYGOMIS
SPRENDIMAS BAIGTINIŲ SKIRTUMŲ METODU

Daktaro disertacijos santrauka

Fiziniai mokslai (P 000),
Matematika (01 P)

Justina JACHIMAVIČIENĖ

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WITH NONLOCAL INTEGRAL CONDITIONS
BY FINITE DIFFERENCE METHOD

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