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INVESTIGATION OF STOCHASTIC LINEAR PROGRAMMING BY MONTE CARLO METHOD

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VYTAUTO DIDŽIOJO UNIVERSITETAS
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STOCHASTINIO TIESINIO PROGRAMAVIMO MONTE KARLO METODU TYRIMAS

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Research field

In tasks of resource and finance planning, job scheduling management, various problems with non deterministic parameters and various kind of uncertainty are often being faced. This uncertainty often is described by statistical probabilistic methods. These tasks are solving by stochastic linear and non-linear methods. Two-stage or multistage stochastic linear problems are extension of the classic linear programming, when parameters of the problem may be random variables.

Research field of the present work is research of the stochastic non-linear problems, investigation and applying of Monte Carlo method for developing the algorithms for two-stage stochastic linear programming.

Relevance of the problem

Stochastic programming was developed at second half of 19th century by demand to solve technical, economical and financial problems. Linear programming problems couldn't evaluate uncertainty of planning parameters. Usage of random parameters in linear programming models leads to complicated nonlinear optimization problems, which usually couldn't be solved by direct nonlinear programming methods. Stochastic methods for solving stochastic problems must be developed and applied. These stochastic methods generalize deterministic linear and nonlinear programming methods.

Main problems for stochastic programming are complicated computation of precise values of the objective function and verification of the optimality of the solution.

Then linear stochastic programming tasks are solving, the process of the solving must be selected before some values of the parameters become known, and these values are correcting on the later stage of the process. These values of the parameters may weight the process of the solving or to create different process of the solution.

Solving of the stochastic linear problems under admissible accuracy is actual and imperfect investigated theoretical and practical problem.

Object of the research

The object of research in the present dissertation are stochastic linear programming tasks, two-stage stochastic linear problem, application of Monte Carlo method for stochastic differentiation and optimisation, application of Monte Carlo method to verify statistical hypotheses.

Aim and objectives of the research

Aim of this work is to analyse known algorithms for stochastic linear programming methods, to investigate methods for the estimation and projection of the stochastic gradient and to develop iterate algorithm and software for solving of the stochastic linear problems by Monte Carlo method at acquired accuracy.
In order to achieve this aim, the following objectives were to be solved:

- To investigate methods of the stochastic non-linear gradient optimisation;
- To develop and investigate methods for stochastic differentiation and stochastic gradient estimation;
- To develop and investigate methods for stochastic gradient ε-projection;
- To do computer study for comparison of the various methods of stochastic differentiation;
- To develop the algorithm for stochastic linear programming under ε-admissible directions by Monte Carlo method;
- To develop the algorithm to choose size of Monte Carlo sample;
- To develop the algorithm to verify hypothesis of the optimality by statistical criteria;
- To develop methodology to investigate efficiency of the algorithms for stochastic linear programming by computer study;
- To study numerically convergence of the developed algorithm by solving tasks from the standard database;
- To compare efficiency of the developed algorithm with Dantzig - Wolfe and Benders decomposition methods.

Scientific novelty

There were suggested algorithms for stochastic differentiation:

- Algorithm for solving of the dual linear problem;
- Finite difference algorithm;
- Simulated perturbation stochastic approximation (SPSA) algorithm;
- Likelihood ratio algorithm.

The algorithm of stochastic gradient ε – projection was suggested.

The algorithm of stochastic linear optimisation by Monte Carlo samples series was suggested.

The adjustment rule of Monte Carlo sample size was proposed and investigated. The rule guarantees the convergence a. s. at a linear rate and uses computational resources rationally.

The algorithm for the treatment of optimum hypotheses in a statistical manner was suggested.

It was shown by computer study that developed algorithm makes it possible to solve the two-stage stochastic linear problems as well as applied tasks with sufficient accuracy by means of an acceptable size of computations and computational resources.

Practical significance

- The software, by means of Delphi, FreePascal, MathCAD, C++, has been developed, implementing stochastic differentiation algorithm by Monte Carlo method.
- The software, by means of Delphi, FreePascal, MathCAD, C++, has been developed, implementing two-stage stochastic linear programming algorithm by Monte Carlo method.
- The algorithm was applied for solution of the applied problems:
  Manpower planning;
  Power plant investment planning.

Approbation and publications of the research

The research results were presented and discussed at the following national and international conferences in Lithuania:

- "The Algorithm of Stochastic Linear Optimisation by Monte Carlo Samples Series". Conference "Information technologies'2005" in Kaunas University of Technology, 2005, January.
- "The Algorithm of Stochastic Linear Optimization by Monte Carlo Samples Series ". Conference "Mathematics and mathematical modeling" in Kaunas University of Technology, 2005, April.
- "Nonlinear Stochastic Programming by Monte Carlo estimators". 12th conference of LIKS (Lithuanian Computer Society) "Computer Days - 2005" in Klaipeda University, 2005, September.
- "Application of the Monte-Caro Method to Stochastic Linear Programming". Lithuanian young scientists INYS seminar "Optimal Process Design" in Vilnius institute of Mathematics and Informatics, 2006, February.

- "Statistical Criteria for Termination Procedure in Stochastic Linear Programming". Lithuanian young scientists conference „Operation Research and Application“, LOTD-2006, Vilnius, 2006, May.
- "Stochastic Linear Programming by Decomposition Method ". Lithuanian young scientists conference „Operation Research and Application“, LOTD-2007, Vilnius, 2007, May.

The main results of this dissertation were published in **5** scientific journals: 2 **articles** in periodical scientific publications from the list approved by the Science Council of Lithuania; **2 articles** in other reviewed scientific publications, 1 **article** in the proceedings of scientific conferences.

Structure and size of the work

The work is written in Lithuanian. It consists of 6 chapters, the list of references, and appendixes. There are 104 pages of the text, 10 tables, 21 figures, and 120 bibliographical sources.

CONTENT OF THE DISSERTATION

Chapter 1 is introductory and describes the relevance of the problem, the scientific novelty of the results and their practical significance, and the objectives and tasks of the work are formulated.

Chapter 2 analyses the algorithms for stochastic linear programming. Stochastic linear programming is a linear programming where some parameters are probabilistic (random variables with known distribution).

In general stochastic linear problems are formulating as stochastic dynamic programming tasks. A most known and applicable stochastic linear programming task is two-stage stochastic linear problem. If uncertainty in this problem is described by continuous distribution it may be formulated as non-linear programming problem with linear restrictions.

Statements of stochastic linear and non-linear programming tasks are discussed. Mostly applied direct and oblique methods for stochastic programming, Dantzig - Wolfe and Benders decomposition methods are reviewed.

Some stochastic linear programming applied tasks as logistics, manufacture planning, supply chain management etc. are overlooked.

Stochastic optimisation methods – stochastic quasi gradient projection, stochastic approximation, Monte Carlo method - are discussed.

Chapter 3 dedicated to development of two-stage stochastic linear programming method by Monte Carlo method.

It was considered a two-stage stochastic optimisation problem with complete recourse:

$$F(x) = c \cdot x + E\left\{Q(x,\xi)\right\} \to \min_{x \in D \subset \Re_+^n} \quad (1)$$

subject to the feasible set

$$D = \left\{x \middle| \; A \cdot x = b, \; x \in \Re_+^n \right\} \quad (2)$$

where

$$Q(x,\xi) = \min_y [q \cdot y \,|\, W \cdot y + T \cdot x \le h, \; y \in R_+^m] \quad (3)$$

the vectors b, q, h and full rank matrices A, W, T were of the appropriate dimensionality. It was considered the feasible set D been nonempty and bounded. It was assumed vectors q, h and matrices W, T been random in general, and, consequently, depending on an elementary event $\xi \in \Omega$ from certain probability space (Ω, Σ, P). Thus, under uncertainty the modelled system operates in an environment in which there were uncontrollable parameters, which were modelled using random variables. Hence, the performance of such a system could also be viewed as a random variable. It was assumed the measure P been absolutely continuous and defined by the probability density function $p(x,\cdot): \Re^n \times \Omega \to \Re_+$, depending on decision variable x in general. Besides, it was assumed that a solution of the second stage problem (3) and values of function Q almost surely (a. s.) existed and were bounded.

First, by duality of linear programming one had that

$$F(x) = c \cdot x + E\left\{\max_u \left[(h - T \cdot x) \cdot u \mid u \cdot W^T + q \ge 0, \; u \in \Re_+^n \right]\right\}. \quad (4)$$

It could be derived, that under the assumption on the existence of a solution to the second stage problem in (3) and continuity of measure P, the objective function (4) was smoothly differentiable and its gradient was expressed as

$$\nabla_x F(x) = E(g(x,\omega)), \quad (5)$$

where $g(x,\omega) = c - T \cdot u^*$ is given by the a set of solutions of the dual problem

$$(h - T \cdot x)^T \cdot u^* = \max_u [(h - T \cdot x)^T \cdot u \mid u \cdot W^T + q \ge 0, \quad u \in \Re^m],$$

(details were given in Rubinstein and Shapiro (1993), Shapiro (2000), etc.)

Monte-Carlo samples

In solving problem (1), suppose it was possible to get finite sequences of realizations (trials) of ω at any point x and the corresponding solutions of problem (3), and the values of $Q(x,\omega)$ as well as solution of the second stage problem in (3) were available for these realizations. Then it was not difficult to find the Monte-Carlo estimators corresponding to the expectations in (1), (4), (5). Thus, it was assumed that the Monte-Carlo samples of a certain size N were provided for any $x \in R^n$:

$$Y = (y^1, y^2, ..., y^N), \quad (6)$$

where y^i were independent random variables, identically distributed with the density $p(\cdot): \Omega \to R_+^n$, and the sampling estimators were computed:

$$\tilde{F}(x) = \frac{1}{N}\sum_{j=1}^{N} f(x, y^j), \quad (7)$$

$$\tilde{D}^2(x) = \frac{1}{N-1}\sum_{j=1}^{N}(f(x,y^j) - \tilde{F}(x))^2. \quad (8)$$

The estimate of a gradient:

$$\tilde{G}(x) = \frac{1}{N}\sum_{j=1}^{N} g(x, y^j), \quad (9)$$

and the sampling covariance matrix

$$Z(x) = \frac{1}{N-n}\sum_{j=1}^{N}(g(x,y^j) - \tilde{G}) \cdot (g(x,y^j) - \tilde{G})' \quad (10)$$

was used, too.

Stochastic procedure for optimisation

The gradient search approach with projection to the feasible set would be a chance to create the optimising sequence; however, the problems of "jamming" or "zigzagging" were typical in such a case. To avoid them, the $\varepsilon-$ feasible direction approach was implemented.

It was defined the set of *feasible directions* as follows:

$$V(x) = \left\{g \in \Re^n \middle| Ag = 0, \forall_{1 \le i \le n}\left(g_j \le 0, \; if \; x_j = 0\right)\right\}, \; x \in D. \quad (11)$$

It was denoted, g_U as projection of vector g onto the set U. Since the objective function was differentiable, the solution $x \in D$ was optimal if (Bertsekas, 1982):

$$\nabla F(x)_V = 0. \quad (12)$$

It was assumed a certain multiplier $\hat{\rho} > 0$ to be given. It was the function $\rho_x : V(x) \to \Re_+$ defined by

$$\rho_x(g) = \min\left\{\hat{\rho},\ \min_{\substack{g_j>0,\\ 1\le j\le n}}\left(\frac{x_j}{g_j}\right)\right\},\ \exists_{1\le j\le n}\left(g_j > 0\right), \quad (13)$$

$\rho_x(g) = \hat{\rho}$, if $\forall_{1\le j\le n}(g_j \le 0)$. Thus $\left(x + \rho \cdot g\right) \in D$, when $\rho = \rho_x(g)$, for any $g \in V\left(x\right), x \in D$, a certain small value $\hat{\varepsilon} > 0$ were be given. Then the function $\varepsilon_x : V(x) \to \Re_+$ was introduced:

$$\varepsilon_x(g) = \hat{\varepsilon} \cdot \max_{\substack{1\le j\le n\\ g_j>0}}\left\{\min\left\{x_j, \hat{\rho}\cdot g_j\right\}\right\},\ \exists_{1\le j\le n}\left(g_j > 0\right),$$

$\varepsilon_x(g) = 0$, if $\forall_{1\le j\le n}(g_j \le 0)$, and the ε *-feasible set* was defined:

$$V_\varepsilon(x) = \left\{g \in \Re^n \,\middle|\, Ag = 0, \forall_{1\le i\le n}\left(g_j \le 0,\ if\ \left(0 \le x_j \le \varepsilon_x(g)\right)\right)\right\} \quad (14)$$

The stochastic optimisation procedure was developed. Some initial point $x^0 \in D$ were be given, random sample (7) of a certain initial size N^0 be generated at this point, and Monte-Carlo estimates (8), (9), (10), (11) be computed.

For instance, the starting point could be obtained as the solution of the deterministic linear problem:

$$(x^0, y^0) = \arg\min_{x,y}[c\cdot x + q\cdot y \mid A\cdot x = b, W\cdot y + T\cdot x \le h,\ \ y \in R_+^m, x \in R_+^n]. \quad (15)$$

The iterative stochastic procedure of gradient search could be used further:

$$x^{t+1} = x^t - \rho^t \cdot \tilde{G}(x^t), \quad (16)$$

where $\rho^t = \rho_{x^t}(\tilde{G}^t)$ was the step-length multiplier defined by (13), and $\tilde{G}_\varepsilon^t = \tilde{G}(x^t)_{V_\varepsilon(x^t)}$ was the projection of gradient estimator to the ε -feasible set.

The choice of the Monte-Carlo sample size was considered more in detail. Note, that there was no a great necessity to compute estimators with a high accuracy on starting the optimisation, because then it sufficed only to approximately evaluate the direction leading to the optimum. Therefore, one could obtain not so large samples at the beginning of the optimum search and, later on, increased the size of samples so as to get the estimate of the objective function with a desired accuracy just at the time of decision making on finding the solution to the optimisation problem. It was pursued this purpose by choosing the sample size at next iteration inversely proportional to the square of the gradient estimator from the current iteration:

$$N^{t+1} \ge \frac{\hat{\rho}\cdot C}{\rho^t \cdot \left|\tilde{G}_\varepsilon^t\right|^2}, \quad (17)$$

where $C > 0$ was a certain constant. On the other hand, such a rule enables us to ensure the condition of proportionality of stochastic gradient variance to the square of the gradient norm, which is sufficient for convergence. Thus, under certain wide conditions on existence of expectations of estimators such the rule guaranteed the convergence a. s. to optimal solution, i.e., starting from any initial approximation $x^0 \in D$ and $N^0 > 1$, formulae (15), (16), (17) defined the sequence $\left\{x^t, N^t\right\}_0^\infty$ so that $x^t \in D$, and there existed values $\bar{\rho} > 0$, $\varepsilon_0 > 0$, $\bar{C} > 0$ such that

$$\lim_{t\to\infty}\left|\nabla F(x^t)_{V_\varepsilon(x^t)}\right|^2 = 0 \text{ (mod (P))}, \quad (18)$$

For $0 < \hat{\rho} \le \bar{\rho}$, $0 < \varepsilon < 1$, $C \ge \bar{C}$. The proof was given in (Sakalauskas, 2004).

A choice of parameters of the method was discussed. The step length ρ in (24) could be determined experimentally. The choice of constant C or that of the best metrics for computing the stochastic gradient norm in (16) required a separate study. For instance, the choice $C = n\cdot Fish(\gamma, n, N^t - n) \approx \chi_\gamma^2(n)$, where $Fish(\gamma, n, N^t - n)$ was the γ -quantile of the Fisher distribution with $(n, N^t - n)$ degrees of freedom, and estimation of the gradient norm in a metric induced by the sampling covariance matrix (10), ensured that a random error of the stochastic gradient didn't exceed the gradient norm approximately with probability $1-\gamma$. Thus, a following version of (17) for regulating the sample size was proposed in practice:

$$N^{t+1} = \min\left(\max\left(\left[\frac{n\cdot Fish(\gamma, n, N^t - n)}{\rho^t\cdot(\tilde{G}(x^t)\cdot(Z(x^t))^{-1}\cdot(\tilde{G}(x^t))}\right] + n, N_{\min}\right), N_{\max}\right), \quad (19)$$

where minimal N_{min} (usually ~20-100) and maximal N_{max} (usually ~ 10000-20000) values were introduced to avoid great fluctuations of sample size in iterations. N_{max} also may be chosen from the conditions on the permissible confidence interval of estimates of the objective function.

Statistical testing of the optimality hypothesis

A possible decision on finding of optimal solution should be examined at each step of the optimisation process. Since only the Monte-Carlo estimates of the objective function and that of its gradient were known, only the statistical optimality hypothesis could were tested. As far as the stochastic error of these estimates depends in essence on the Monte-Carlo samples size, a possible optimal decision could be made, if, first, there was no reason to reject the hypothesis of equality to zero of the gradient, and, second, the sample size was sufficient to estimate the objective function with the desired accuracy.

Note that the distribution of sampling averages (7) and (9) could be approximated by the one- and multidimensional Gaussian laws (see, e.g., (Bhattacharya and Ranga Rao, 1976). Therefore it was convenient to test the validity of the stationary condition (12) by means of the well-known multidimensional Hotelling T^2-statistics (Krishnaiah and Lee, (1980)). Hence, the optimality hypothesis could be accepted for some point x^t with significance $1-\mu$, if the following condition is satisfied:

$$(N^t - n)\cdot(\tilde{G}(x^t))\cdot(Z(x^t))^{-1}\cdot(\tilde{G}(x^t))/n \le Fish(\mu, n, N^t - n)\,. \qquad (20)$$

Next, the asymptotic normality was used again and decided that the objective function was estimated with a permissible accuracy δ, if its confidence bound didn't exceed this value:

$$\frac{2\cdot\eta_\beta\cdot\tilde{D}(x^t)}{\sqrt{N^t}} \le \delta\,, \qquad (21)$$

where η_β was the β -quantile of the standard normal distribution. Thus, the procedure (11) was iterated adjusting the sample size according to (19) and testing conditions (20) and (21) at each iteration. If the latter conditions were met at some iteration, then there were no reasons to reject the hypothesis on the optimum finding. Therefore, there was a basis to terminate the optimisation and make a decision on the optimum finding with a permissible accuracy. If at least one condition out of (20), (21) was unsatisfied, then the next sample was generated and the optimisation was continued. The optimisation should terminate after generating a finite number of Monte-Carlo samples.

The algorithms were presented:

- algorithm of solution or the two-stage stochastic linear problem;
- algorithms for two-stage stochastic linear problem objective, gradient estimators and for covariance matrix;
- algorithm for vector projection to the set;
- algorithm for vector ε-projection;

- algorithm for Hotelling T^2 statistics;
- algorithm for the length of the gradient search step.

Chapter 4 is dedicated to investigation of developed algorithms.

Investigation of the estimator of the stochastic gradient

Four methods of stochastic differentiation were studied:

- analytical approach (differentiation of the integral) (AA),
- finite difference (FD),
- simulated perturbation stochastic approximation (SPSA),
- likelihood ratio (LR).

It was assumed the randomness be exogenous and no effected by decision variable, i.e. density $p(\cdot)$ didn't depend on variable *x*.

At the AA approach the formula

$$g(x,y) = \nabla_x f(x,y)$$

was used.

At the FD approach the each i[th] component of the stochastic gradient $g^2(x,\xi)$ was computed as:

$$g_i^2(x,y) = \frac{f(x+\delta\cdot\varsigma_i, y) - f(x,y)}{\delta}\,,$$

ς_i was the vector with zero components except i[th] one, equal to 1, δ was some small value.

At the SPSA approach additional function value computation was examined:

$$g^3(x,y) = \frac{f(x+\delta\cdot\nu, y) - f(x-\delta\cdot\nu, y)}{2\cdot\delta}\,,$$

where ν was the random vector obtaining values 1 or -1 with probabilities p=0.5 (see Spall, 1992), δ was some small value.

At the LR approach evaluation of stochastic gradient was made by using formula:

$$g^4(x,y) = \frac{\left(f\left(x+d\cdot y\right) - f\left(x+d\cdot E\xi\right)\right)\cdot y}{d}\,,$$

if $f(x,y) = f(x+d\cdot y)$, $y \sim N(0, d\cdot I_n)$.

Computer simulation study of gradient estimators was presented using testing example.

The function

$$F(x) \equiv Ef_0(x+\xi)$$

being expectation of

$$f_0(y) = \sum_{i=1}^{n}(a_i y_i^2 + b_i \cdot (1-\cos(c_i \cdot y_i)))$$

where y_i were random and normally $N(0,d^2)$ distributed, d=0.5, a_i were uniformly distributed in [2, 5], b_i - in [1, 2] and c_i - in [-0.5, 0.5] and $2 \le n \le 100$.

Due to symmetry this function has the minimum at the point $x_+ = 0$. Thus, 400 Monte-Carlo samples of size N = (1-10, 20, 40, 60, 80, 100) were generated at this point and the T^2-statistic in optimal criterion was computed for each sample using various stochastic gradients. The hypothesis on the difference of empirical distribution of this statistic from the Fisher distribution was tested according to the criteria ω^2 and Ω^2. Values of ω^2 and Ω^2 statistics computation for AA estimator on variable number and sample size are given in Table 1 and Table 2. The critical value ω^2=0.46 (p=0.05), and that of the next one is Ω^2=2.49 (p=0.05). Values of statistics exceeding critical value are bolded. Thus the minimal size of the Monte Carlo sample necessary to approximate the distribution of Hotelling statistics by Fisher distribution depended on dimensionality of the task n.

Table 1. ω^2 criteria results by variable number and sample size

n \ N	**50**	**100**	**200**	**500**	**1000**
2	0.30	0.24	0.10	0.08	0.04
3	0.37	0.12	0.09	0.06	0.04
4	0.19	0.19	0.13	0.08	0.04
5	**0.75**	0.13	0.12	0.08	0.06
6	**1.53**	0.34	0.10	0.10	0.08
7	**1.56**	0.39	0.13	0.08	0.09
8	**1.81**	0.42	0.27	0.18	0.10
9	**4.18**	**0.46**	0.26	0.20	0.12
10	**8.12**	**0.56**	**0.53**	0.25	0.17

Table 2. Ω^2 criteria results by variable number and sample size

n \ N	**50**	**100**	**200**	**500**	**1000**
2	**2.57**	1.14	0.66	0.65	0.42
3	**2.78**	0.82	0.65	0.60	0.27
4	**3.75**	1.17	0.79	0.53	0.31
5	**4.34**	1.46	0.85	0.64	0.36
6	**8.31**	2.34	0.79	0.79	0.76
7	**8.14**	**2.72**	1.04	0.52	0.45
8	**10.22**	**2.55**	1.87	0.89	0.52
9	**20.86**	**2.59**	1.57	1.41	0.78
10	**40.57**	**3.69**	**3.51**	1.56	0.98

This requiring Monte-Carlo sample size depending on the dimensionality (for confidence α = 0.95) is given in the Table 3. Similar results are obtained for other estimators, too.

Table 3. Requiring Monte-Carlo sample size by number of variables

Number of variables	Monte Carlo sample size
10	100
20	1000
40	2200
60	3300
80	4500
100	6000

As follows from simulation results the distribution of Hotelling statistics can be approximated by Fisher distribution appropriately choosing sample size.

Investigation of the reliability of the criteria of the optimality hypothesis

Further the dependencies of the frequency of optimality hypothesis (equality of gradient to zero) according to criterion on the distance $r = |x - x^+|$ to the optimal point and Monte Carlo sample size N for various gradient estimators were studied. The purpose of such study was to answer how estimators used are good to reject the hypothesis of optimality in the point of solution which differs from optimal one. These dependencies for n=2 and more variables are presented in Fig. 1-5 (for confidence α =0.95).

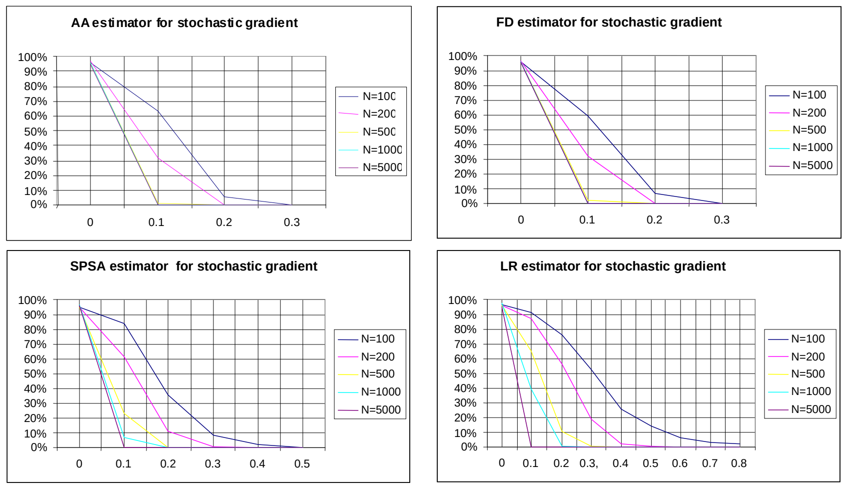


Fig. 1. Frequency of optimality hypothesis (n=2).

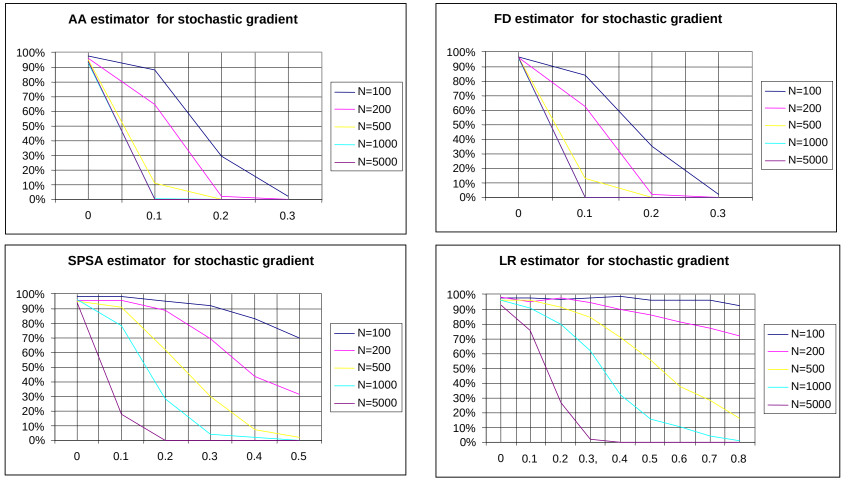


Fig. 2. Frequency of optimality hypothesis (n= 10).

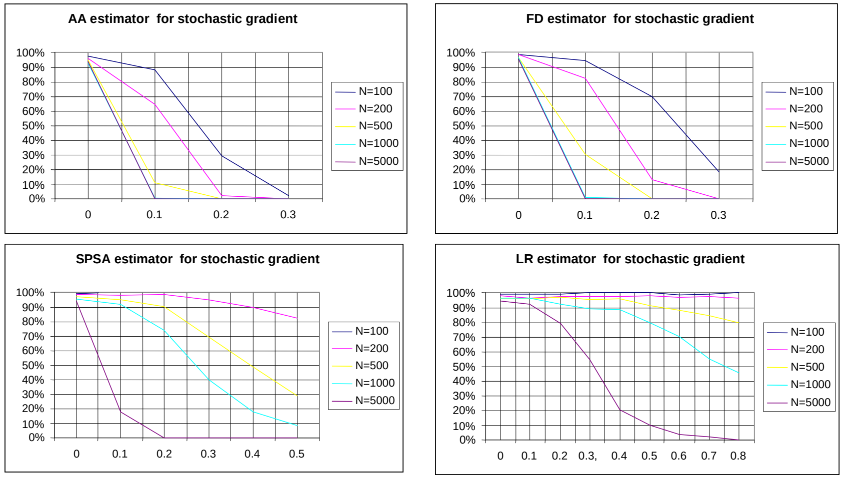


Fig. 3. Frequency of optimality hypothesis (n=20).

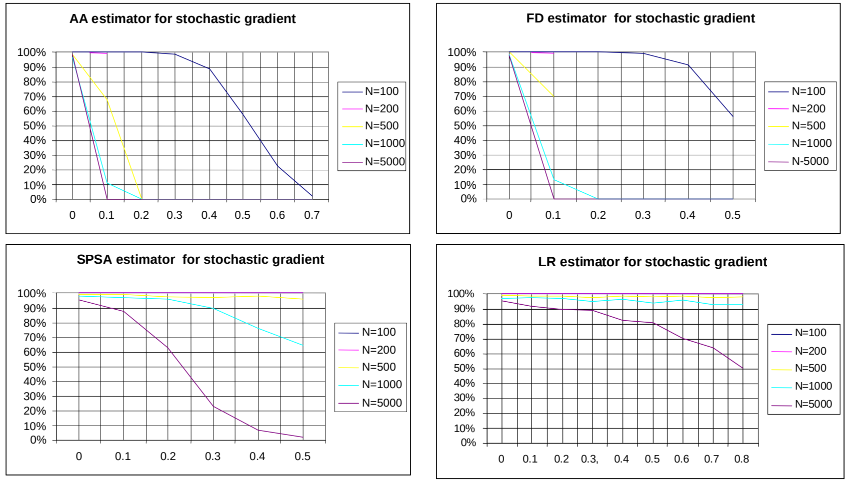


Fig. 4. Frequency of optimality hypothesis (n=50).

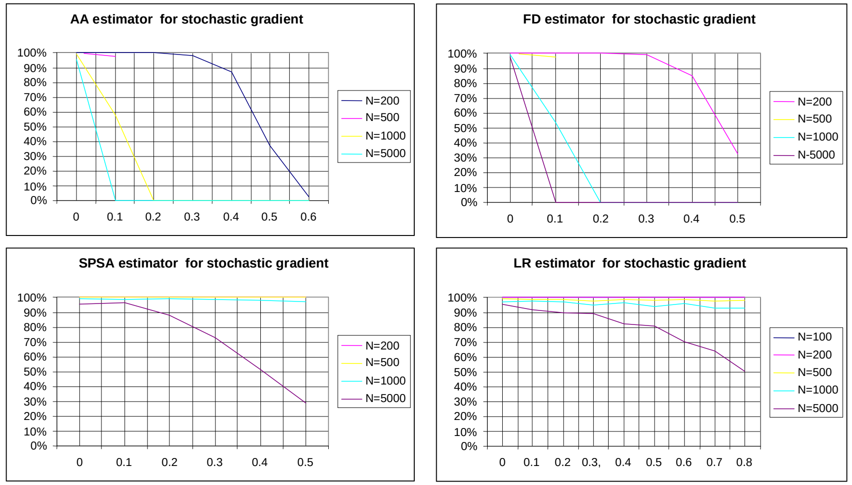


Fig. 5. Frequency of optimality hypothesis (n=100).

Thus, the computation results show that AA and FD approach provides estimators for a reliable checking of optimality hypothesis in a wide range of dimensionality of stochastic optimisation problem ($2 \le n \le 100$). However, SPSA and LR estimators can be applied for stochastic gradient estimation only for tasks of not very large dimensionality: $1 \le n \le 20$.

Two-stage stochastic linear optimisation problems

Data of the two-stage stochastic linear optimisation problem was taken from a database at the address http://www.math.bme.hu/~deak/twostage/ (accessed 20/01/2006). Dimensions of the tasks were as follows: the first stage had n_1 rows and m_1 variables; the second stage had n_2 rows and m_2 variables.

Dimensions of the 1st task were as follows: the first stage had 10 rows and 20 variables; the second stage had 20 rows and 30 variables.

The estimate of the optimal value of the objective function given in the database was 182.942 ± 0.066. Application of the approach considered allowed to improve the estimate of the optimal value up to 182.592 ± 0.033.

The results, obtained in solving this task 400 times by the method (15), (16), (19) were considered. Initial data were as follows $\gamma = \beta = 0.95$, $\mu = 0.99$, $\varepsilon = 0.1; 0.2; 0.5; 1.0$, $N^0 = N_{min} = 100$, maximal number of iterations $t_{\max} = 100$, generation of trials was broken when the estimated confidence interval of the objective function exceeds admissible value ε.

Termination conditions were satisfied at least once time for all paths of optimisation. Thus, the conclusion on the optimum finding with an admissible accuracy could be made for all paths (the sampling frequency of termination after t iterations with confidence intervals is presented in Fig. 6). The averaged dependencies of the sample size, the objective function, the confidence interval of (21), and Hotelling statistics of (20) by the iteration number t are given in Fig's. 7 – 10 to illustrate the convergence and the behaviour of the optimisation process. In Fig. 11 is a histogram of the ratio $\sum_{j=1}^{t} \frac{N^j}{N^t}$ is depicted.

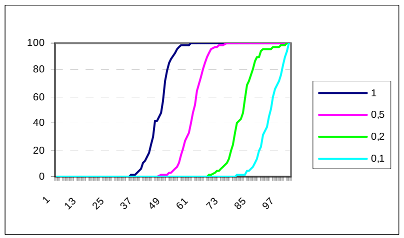


Fig. 6. Frequency of stopping under admissible interval.

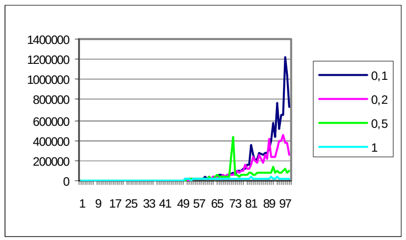


Fig. 7. Change of the sample size under admissible interval.

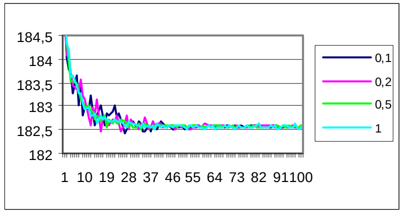


Fig. 8. Change of the objective function under admissible interval.

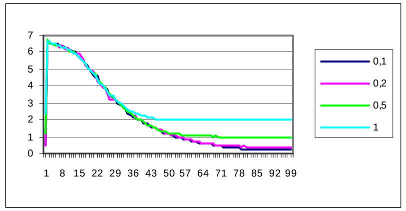


Fig. 9. Change of confidence interval under admissible interval.

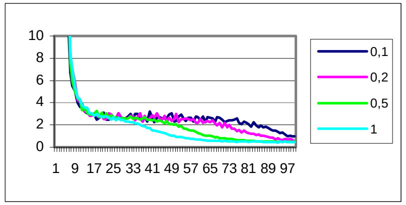


Fig. 10. Change of the Hotelling statistics under admissible interval.

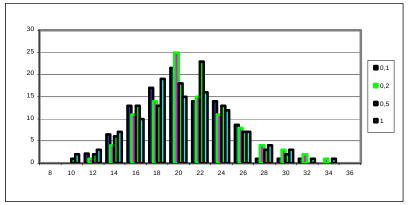


Fig. 11. Histogram of ratio $\sum_{j=1}^{t} \frac{N^j}{N^t}$ under admissible interval.

Table 4. Value of the objective function F and ratio $\sum_{j=1}^{t} \frac{N^j}{N^t}$ under admissible interval ε.

Estimated confidence interval, ε	Value of the objective function, F	Ratio, $\sum_{j=1}^{t} \frac{N^j}{N^t}$
0.1	182.6101	20.14
0.2	182.6248	19.73
0.5	182.7186	19.46
1	182.9475	19.43

Dimensions of the 2nd task were as follows: the first stage had 10 rows and 20 variables; the second stage had 20 rows and 30 variables.

The estimate of the optimal value of the objective function given in the database was 266.683 ± 0.187. Application of the approach considered allowed to improve the estimate of the optimal value up to 266.227 ± 0.066.

Dimensions of the 3rd task were as follows: the first stage had 30 rows and 60 variables; the second stage had 60 rows and 90 variables.

The estimate of the optimal value of the objective function given in the database was 300.841 ± 0.039. Application of the approach considered allowed to improve the estimate of the optimal value up to 300.668 ± 0.033.

Dimensions of the 4th task were as follows: the first stage had 40 rows and 80 variables; the second stage had 80 rows and 120 variables.

The estimate of the optimal value of the objective function given in the database was 586.329 ± 0.327. Application of the approach considered allowed to improve the estimate of the optimal value up to 475.012 ± 0.999.

Comparison of various methods

The 1st task was solved by Dantzig-Wolf method, too. The task was formulated as a large deterministic linear problem with various (5, 10, 20, 30, 40) amounts of the samples of the vector h series. These problems were solved 400 times with various vectors h series and the results were averaged.

Results of the task solution by Dantzig - Wolfe method are shown in Fig. 12 and Fig. 13.

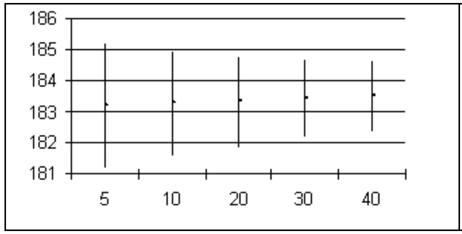


Fig. 12. Objective and confidence interval under amount of samples.

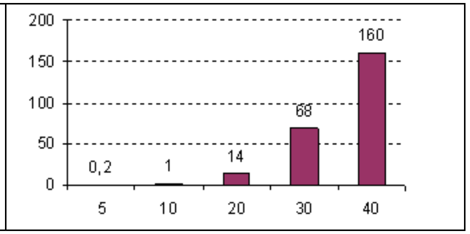


Fig. 13. Average time (in seconds) of the solution under amount of samples.

Results of the task solution by Monte Carlo method are shown in Fig. 14 and Fig. 15.

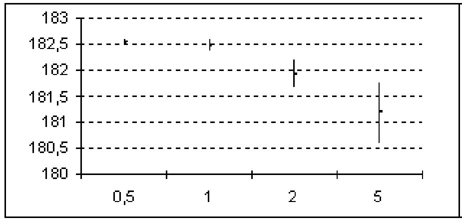


Fig. 14. Objective and confidence interval under admissible interval.

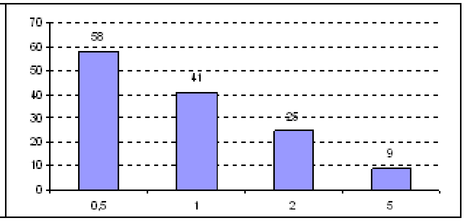


Fig. 15. Average time (in seconds) of the solution under admissible interval.

Comparison of testing results is shown in Fig. 16.

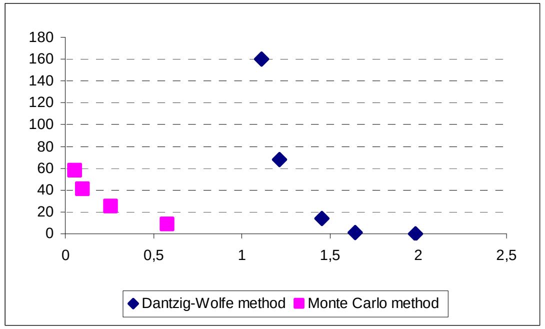


Fig. 16. Average time (in seconds) of the solution under confidence interval of objective.

Testing results showed that the developed algorithm by finite Monte Carlo sample series were allowed effective solving of two-stage linear stochastic problem and sometimes allowed obtaining better results.

Two applied problems – manpower planning and power plant investment planning – were solved by developed algorithm.

Manpower Planning Problem

The manpower-planning problem was considered (Ermolyev and Wets, 1988), where the employer must decide upon a base level of regular staff at various skill levels. The recourse actions available were regular staff overtime or outside temporary help in order to meet unknown demand for service at the minimal cost. The problem was as follows: choose $x = (x_1, x_2, x_3)$ to minimize

$$F(x,z) = \sum_{j=1}^{3} c_j \cdot x_j + \sum_{t=1}^{12} E \min\left(\sum_{j=1}^{3} (q_j \cdot y_{j,t} + r_j \cdot z_{j,t}) \right)$$

subject to

$$x_j \geq 0,\ y_{j,t} \geq 0,\ z_{j,t} \geq 0,$$

$$\sum_{j=1}^{3} \left(y_{j,t} + z_{j,t} \right) \geq w_t - \alpha_t \cdot \sum_{j=1}^{3} x_j,\ t = 1,2,\ldots,12,$$

$$y_{j,t} \leq 0.2 \cdot \alpha_t x_j,\ j = 1,2,3,\ \ t = 1,2,\ldots,12,$$

$$\gamma_{j-1}\left(x_j + y_{j-1,t} + z_{j-1,t}\right) - \left(x_j + y_{j-1,t} + z_{j-1,t}\right) \geq 0,$$

$$j = 1,2,3,\ \ t = 1,2,\ldots,12,$$

where

x_j base level of regular staff at the skill level j = 1, 2, 3,

$y_{j,t}$ amount of overtime help,

$z_{j,t}$ amount of temporary help,

c_j cost of regular staff at the skill level j = 1, 2, 3,

q_j cost of overtime,

r_j cost of temporary help,

w_t demand for services at the period *t*,

α_t anticipated absentees rate for regular staff at time *t*,

γ_{j-1} ratio of the amount of skill level *j* per amount of *j-1* required,

η variation of random demand for services 0, 1, 10, 20, 30.

The demands w^t were independent normal: $N(\mu, \sigma^2)$, where $\mu_t = \eta \cdot \sigma_t^2$. Initial data and other details could be found in (Ermolyev and Wets, 1988). This problem had 3 variables at the first stage and 72 linear inequalities with 75 variables at the second stage.

The results of solving manpower problem were considered. Optimal solutions of solving this task varying variation η are given in Table 5 (costs of manpower are given by USD, the admissible confidence interval of the objective function is 0.1 (100 USD)).

Table 5. Manpower amount on levels and cost (in dependence of variation η).

	Base level of regular staff			Cost of manpower (confidence interval 0.1)
η	X_1	X_2	X_3	F
0	9222	5533	1106	94.899
1	9222	5533	1106	94.899
10	9376	5616	1106	96.832

It was proved (Sakalauskas, 2000, 2002) that the approach considered ensures the linear rate of convergence. It followed from linearity of this rate

that the total amount of Monte-Carlo trials $\sum_{j=1}^{t} N^j$ performed to get the optimal solution was approximately proportional to the amount of trials N^t necessary to solve the problem with admissible accuracy. Besides, the ratio $\sum_{j=1}^{t} \frac{N^j}{N^t}$ was determined mostly by the positive definiteness of Hessian of the objective function and almost didn't depend on the admissible accuracy ε (details in Sakalauskas, 2002). The Table 6 illustrates this fact where this ratio is presented for various ε and η**.** Thus, the conclusion followed that if one had a certain resource to compute one value of the objective function with an admissible accuracy, then the optimisation required only several times more computations. This enabled to construct reasonable, from a computational viewpoint, stochastic methods for stochastic programming with admissible accuracy.

Table 6. Ratio $\sum_{j=1}^{t} \frac{N^j}{N^t}$ under admissible interval ε and variation η.

ε	0,05	0,1	0,2
η=10	16.1	10.7	10.9
η=30	21.4	21.3	20.2

The averaged dependencies of the objective function, of the confidence interval, of the statistic $\frac{T_t^2}{Fish(\mu, n, N^t - n)}$ and of the sample size by the iteration number *t* are given in Fig's. 17-20 to illustrate the convergence and the behaviour of the optimisation process, then variation η=10 and admissible accuracy ε=100 USD.

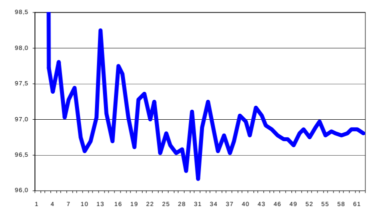


Fig. 17. Dependencies of the objective function under the iteration number *t*.

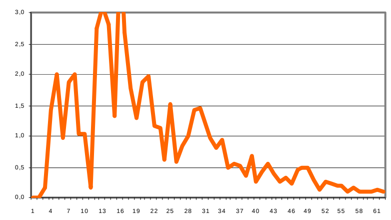
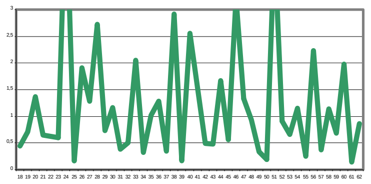


Fig. 18. Dependencies of the confidence interval under the iteration number *t*.



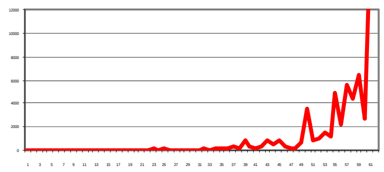


Fig. 19. Dependencies of the statistic under the iteration number *t*.

Fig. 20. Dependencies of the sample size under the iteration number *t*.

Power Plant Investment Planning Problem

An energetic concern must invest in a system of power plants to meet its current and future demand for electrical power. These plants were to be built for the first year only, and were expected to operate over the next 15 years. The budget for construction of power plants was $10 billion, which was to be allocated for four different types of plants: gas turbine, coal, nuclear power, and hydroelectric. The objective was to find the power plant allocation which had minimized the sum of the investment cost and the expected value of the operating cost over 15 years. The operating cost was stochastic due to uncertainty in future demand.

Power plants were priced according to their electric capacity, measured in gigawatts (GW). Table 7 shows the investment cost per GW of capacity for each type of plants.

Table 7. Investment cost per GW of capacity.

Plant	Cost per GW capacity
Gas Turbine	$110 million
Coal	$180 million
Nuclear power	$450 million
Hydroelectric	$950 million

Projected demand for electric power was normally distributed random value D~ $N(\mu,\ 0{,}5)$. A set of demands and durations during the year is shown in Table 8.

Table 8. Power demand during the year.

Demand Block	Demand (μ ,GW)	Duration (hours)
#1	26.0	490
#2	21.5	730
#3	17.3	2190
#4	13.9	3260
#5	11.1	2090

The operating costs for the first year of each type of power plants, as well as the cost of purchasing power from an external source, are shown in Table 9, where the units are in cents per kilowatt-hour (KWH). Each year costs grow with rate 1%.

Since hydroelectric energy depends on the availability of rivers which may be dammed, the geography of the region constrained the hydroelectric power capacity no more than 5.0 GW.

Table 9. Operating cost of power generation.

Plant	Cost per KWH
Gas Turbine	3.92 ¢
Coal	2.44 ¢
Nuclear	1.40 ¢
Hydroelectric	0.40 ¢
External Source	15.0 ¢

The problem could be modelled as a two-stage stochastic linear optimisation model

$$\sum_{i=1}^{4} c_i x_i + E\left\{\min_y \left(\sum_{i=1}^{5}\sum_{j=1}^{5}\sum_{k=1}^{15} q_i h_j r_k y_{ijk\omega}\right)\right\} \to \min,$$

s. t.

$$\sum_{i=1}^{4} c_i x_i \le 10000,$$

$$x_4 \le 5.0,$$

$$y_{ijk\omega} \le x_i,\ \ i = 1,2,3,4,\ \ \forall\, j,k,\omega,$$

$$\sum_{i=1}^{5} y_{ijk\omega} \ge D_{jk\omega},\ \ \forall\, j,k,\omega,$$

$$x \ge 0,\ y \ge 0,$$

where

$x = (x_1, x_2, x_3, x_4)$ – vector, representing the gigawatts of capacity to be built for each type of plant,

$y_{ijk\omega}$ – amount of electricity capacity used to produce electricity by power plant type *i* for demand block *j* in year *k* in GW,

c_i – investment cost per GW of capacity for power plant type *i*,

q_i – operating cost of power generation for power plant type *i*,

h_j – duration of the demand block *j*,

r_k – operating costs growth in year *k*,

$D_{jk\omega}$ – power demand in year *k* at demand block *j* : $N\left(\mu_j,\ 0{,}5\right)$.

Results of the problem solution by Monte Carlo method are shown in Table 10.

Table 10. Power plant capacity construction decisions.

Plant	Optimal Construction Decision based on stochastic data	Known Construction Decision
Gas Turbine	4.66 GW	4.67 GW
Coal	4.57 GW	4.59 GW
Nuclear	4.68 GW	4.67 GW
Hydroelectric	5.00 GW	5.00 GW

The optimal expected cost of power plant investment problem is turns out to be \$16483 ± 29 billion versus known expected cost \$16933 ± 53 billion (Freund, (2004)).

Chapter 5 presents generalization of obtained results, conclusions and recommendations.

The list of references and annexes are presented at the end of the dissertation.

Annexes present solution of the linear programming tasks by Dantzig - Wolfe and Benders decomposition methods, data of the two-stage stochastic linear optimisation problems from a database at the address http://www.math.bme.hu/~deak/twostage/, description of the AMPL (Applied Mathematical Programming Language) and solution of some linear programming tasks by AMPL, relaxation method for decomposition algorithms, method or variables separation for decomposition algorithms.

CONCLUSIONS

The following results were achieved by tackling the objectives posed in the present work:

1. The inerratic the algorithm for stochastic linear programming by Monte Carlo method was developed;
2. The algorithms for stochastic differentiation -for analytical approach (differentiation of the integral), finite difference, simulated perturbation stochastic approximation and likelihood ratio methods, - were developed and investigated.
3. The ε-projection algorithms for stochastic gradient and for regularization of the stochastic gradient.
4. The algorithm to choose size of Monte Carlo sample, that was guarantying the convergence and used computation resources rationally, was developed and investigated;
5. The algorithm to verify hypothesis of the optimality by statistical criteria was developed;
6. The algorithm for solving two-stage stochastic linear programming tasks by finite Monte Carlo samples series was developed;
7. The algorithms and the library of programming modules for Delphi, Free Pascal, MathCAD and C++ by implementing stochastic differentiation and the solving of two-stage linear stochastic programming tasks by Monte Carlo method were developed.
8. The algorithms were applied for practical tasks – manpower planning and power plant investment planning.

On the base of the results obtained and investigations carried out, we can draw the following conclusions:

1. During the calculating experiment, it was shown that developed algorithm for solving two-stage stochastic linear programming tasks by finite Monte Carlo samples series allowed to obtain solutions of the two-stage linear stochastic programming tasks by admissible accuracy during acceptable computer time and memory resources.
2. The results of the computer study showed that the estimator of the stochastic gradient, obtained by solving of the dual task, allowed to confirm the hypothesis that the gradient of the objective function was equal zero by given reliability then number of the variables are less or equal to 100.

3. The developed algorithm allowed solving all problems from the standard database of two-stage linear stochastic programming tasks and improving some of them.
4. The results of statistical modelling validated theoretical conclusions about the rate of the convergence of the developed algorithm. The results showed that total amount of the calculations was straight proportional to amount of the calculations needed to appreciate one value of the objective function by given accuracy.
5. The developed algorithm doesn't require additional resources of the computer memory versus known decomposition methods that are used to solve similar problems.
6. The developed method for stochastic linear programming allows obtain more precisely solutions comparatively to decomposition methods by using less computational resources.

PUBLICATIONS

The research results were published in the following scientific publications:

1. **Sakalauskas L., Žilinskas K.** Application of the Monte Carlo Method to Stochastic Linear Programming. *Series on Computers and Operations Research*, 2006, Vol.7, Computer Aided Methods in Optimal Design and Operations, p. 39-48. (ISBN 981-256-909-7)
2. **Sakalauskas L., Žilinskas K.** Application of Statistical Criteria to Optimality Testing in Stochastic Programming. *Technological and economic development of economy*, 2006, Vol.12, No.4, p. 314-320. (ISSN 1392-8619)
3. **Sakalauskas L., Žilinskas K.** Non-linear Stochastic Programming by Monte Carlo Estimators. *Information Sciences,* 2005, No.34, p. 326-330, in Lithuanian. (ISSN 1392-0561)
4. **Sakalauskas L., Žilinskas K.** The Algorithm of Stochastic Linear Optimisation by Monte Carlo Samples Series. *Mathematics and mathematical modelling*, 2005, Vol.1, p. 20-25, in Lithuanian. (ISSN 1822-2757)
5. **Sakalauskas L., Žilinskas K.** The Algorithm of Stochastic Linear Optimisation by Monte Carlo Samples Series. Proceedings of conference "Information technologies '2005", 2005, Vol.2, p. 425-431, in Lithuanian. (ISBN 9955-09-789-2)

Short description about the author

Kęstutis Žilinskas graduated secondary school of Tyruliai in 1974. In 1981 he received the diploma of higher education in mathematics and physics from Šiauliai Pedagogical Institute, the M.Sc. degree of social sciences in 1994, and the diploma of higher education in informatics from Institute of Qualification in University of Šiauliai. In 2005 he entered upon his doctoral studies at the Institute of Mathematics and Informatics in Vilnius. Research interests: continuous and discrete optimisation, stochastic optimisation, extreme value theory, Monte-Carlo method, optimal design. He is an assistant of Šiauliai University.

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REZIUMĖ

Tyrimų sritis

Sprendžiant išteklių bei finansų planavimo, darbų kalendorinio paskirstymo ir vadybos uždavinius, dažnai susiduriama su problemomis, kuriose parametrai gali būti nedeterminuoti ir susiję su įvairaus pobūdžio neapibrėžtimi. Ši neapibrėžtis dažniausiai yra aprašoma statistiniais tikimybiniais metodais, o minėti uždaviniai yra sprendžiami stochastinio tiesinio arba netiesinio programavimo metodais. Dviejų arba kelių etapų stochastinio tiesinio programavimo uždaviniai yra klasikinio tiesinio programavimo apibendrinimas, kai uždavinio parametrai gali būti atsitiktiniai kintamieji. Šio darbo tyrimų sritis yra stochastinio netiesinio programavimo bei Monte Karlo metodo tyrimas ir taikymas dviejų etapų stochastinio tiesinio programavimo algoritmams sudaryti.

Problemos aktualumas

Stochastinis programavimas, kaip tiesinio programavimo apibendrinimas, atsirado XX a. šeštajame dešimtmetyje dėl techninio, ekonominio ir finansinio planavimo poreikių. Galima paminėti aktualias stochastinio programavimo taikymo sritis:

- elektros energijos gamyba;
- produkcijos gamyba ir transportavimas;
- personalo valdymas;
- logistika;
- investicijų valdymas;
- objektų išdėstymas;
- mechanizmų stabilumas;
- biologinių sistemų analizė ir kt.

Klasikiniuose tiesinio programavimo algoritmuose nebuvo atsižvelgiama į daugelio planavimo parametrų neapibrėžtumą. Sąvoka „stochastinis programavimas“ atsirado kartu su netiesinio programavimo sąvoka, kai matematinio programavimo pradininkai Dancigas, Čarnesas ir Kuperis pradėjo analizuoti tiesinio programavimo uždavinius su atsitiktiniais koeficientais. Atsitiktinių parametrų naudojimas tiesinio programavimo modeliuose veda prie sudėtingų netiesinių ekstreminių uždavinių, kurie dažniausiai negali būti sprendžiami tiesioginiais tiesinio ir netiesinio programavimo metodais.

Pagrindinės stochastinio programavimo uždavinių sprendimo problemos – sudėtingas tikslo funkcijos tikslių reikšmių apskaičiavimas bei gautojo taško

priklausomumo leistinajai sričiai patikrinimas. Stochastiniame programavime reikia rasti uždavinio optimalų sprendinį, kai uždavinio tikslo funkcijos tikslią reikšmę optimaliame taške apskaičiuoti beveik neįmanoma ir kai yra ribojimai, kurių patikrinti taip pat beveik neįmanoma. Sprendžiant tiesinio programavimo uždavinius, sprendimo eiga turi būti pasirinkta dar nežinant kai kurių parametrų reikšmių, kurios yra tikslinamos vėlesniame sprendimo etape. Šios parametrų reikšmės gali turėti įtakos sprendimo eigos pasirinkimui, ar net sukurti skirtingą sprendimo eigą.

Stochastinio tiesinio programavimo uždavinių sprendimas duotu tikslumu yra aktuali ir dar nepakankamai išnagrinėta teorinė ir praktinė problema.

Tyrimų objektas

Disertacijos tyrimų objektas yra tiesioginis ir dualusis stochastinio tiesinio programavimo uždaviniai, dviejų etapų stochastinio tiesinio programavimo uždavinys, Monte Karlo metodo taikymas stochastiniam diferencijavimui ir optimizavimui, Monte Karlo įverčių taikymas statistinėms hipotezėms tikrinti.

Tyrimų tikslas ir uždaviniai

Bendru atveju STP uždaviniai formuluojami kaip stochastinio dinaminio programavimo uždaviniai. Šių uždavinių sprendimas suvedamas į determinuoto didelio matavimo tiesinio programavimo uždavinio, gauto diskretizuojant atsitiktinius parametrus, sprendimą. Tokiu būdu gauto uždavinio ribojimų matrica pasižymi tam tikra struktūra, kuria pasinaudojama kuriant įvairius dekompozicijos algoritmus STP uždaviniams spręsti. Atskiras STP atvejis yra dviejų etapų stochastinio programavimo uždavinys, į kurį gali būti suvedami daugelis taikomųjų uždavinių, kai neapibrėžtis aprašoma tolydiniu tikimybiniu dėsniu. Dekompozicijos metodų, paremtų tolydžiojo tikimybinio dėsnio diskretizavimu, taikymas šiam uždaviniui spręsti susijęs su skaičiavimo problemomis, saugant kompiuterio atmintyje didelius duomenų masyvus, bei gauto sprendinio tikslumo įvertinimu.

Kita kryptis, leidžianti kurti praktinius stochastinio programavimo algoritmus, yra netiesinio stochastinio programavimo ir Monte Karlo metodų taikymas.

Darbo tikslas yra atlikti žinomų stochastinio tiesinio programavimo metodų analitinį tyrimą, ištirti stochastinio gradiento įvertinimo bei projektavimo metodus ir sudaryti iteracinį algoritmą bei programinę įrangą stochastinio tiesinio programavimo uždaviniui spręsti Monte Karlo metodu duotu tikslumu.

Siekiant šio tikslo, darbe sprendžiami tokie uždaviniai:

- Ištirti stochastinio netiesinio gradientinio optimizavimo metodus;
- Sudaryti ir ištirti stochastinio diferencijavimo ir stochastinio gradiento įvertinimo metodus;
- Sudaryti ir ištirti stochastinio gradiento ε -projektavimo metodus;
- Atlikti kompiuterinį eksperimentą įvairiems stochastinio diferencijavimo metodams palyginti;
- Sudaryti stochastinio tiesinio programavimo ε -leistinųjų krypčių algoritmą Monte Karlo metodu;
- Sudaryti algoritmą Monte Karlo imčių tūriui parinkti;
- Sudaryti algoritmą sprendinio optimalumo hipotezei patikrinti statistiniais kriterijais;
- Sudaryti metodiką stochastinio tiesinio programavimo algoritmų efektyvumui tirti kompiuterinio modeliavimo būdu;
- Kompiuterinio modeliavimo būdu ištirti sudaryto algoritmo konvergavimą, sprendžiant standartinės testinių pavyzdžių duomenų bazės uždavinius;
- Palyginti sudaryto algoritmo efektyvumą su Dancigo - Vulfo ir Benderso dekompozicijos metodais.

Mokslinis naujumas

Darbe gauti tokie nauji rezultatai.

Sudaryti ir ištirti stochastinio diferencijavimo algoritmai:

- dualaus uždavinio sprendimo,
- baigtinių skirtumų,
- modeliuojamojo pokyčio,
- tikėtinumo santykio.

Sudarytas stochastinio gradiento ε – projektavimo algoritmas.

Sudarytas dviejų etapų stochastinio tiesinio programavimo iteracinis algoritmas Monte Karlo metodu.

Sudaryta ir ištirta Monte Karlo imčių tūrio parinkimo taisyklė, garantuojanti konvergavimą bei racionaliai panaudojanti skaičiuojamuosius išteklius.

Sudarytas algoritmas optimalumo hipotezei patikrinti, taikant statistinius kriterijus.

Sprendžiant testinius uždavinius parodyta, jog sudarytas algoritmas leidžia surasti uždavinių, panašių į iškylančiu praktikoje, sprendinius reikiamu tikslumu panaudojant priimtinus kompiuterinio laiko ir atminties išteklius.

Praktinė darbo reikšmė

Darbe gauti tokie praktiniai rezultatai:

- Sukurti algoritmai ir programinių modulių biblioteka Delphi, FreePascal, MathCad, C++ priemonėmis, realizuojanti stochastinio diferencijavimo algoritmą Monte Karlo metodu.
- Sukurti algoritmai ir programinių modulių biblioteka Delphi, FreePascal, MathCad, C++ priemonėmis, realizuojanti dviejų etapų stochastinio tiesinio programavimo algoritmą Monte Karlo metodu.
- Sudarytas algoritmas yra pritaikytas taikomiesiems uždaviniams spręsti:
  darbo organizavimo planavimas;
  investavimo ir elektros energijos gavybos planavimas.

Darbo rezultatų aprobavimas ir publikavimas

Tyrimo rezultatai publikuoti **5** moksliniuose leidiniuose: **2** straipsniai leidiniuose, esančiuose Lietuvos mokslo tarybos patvirtintame tarptautinių duomenų bazių sąraše (tarptautinių konferencijų darbų leidiniuose, referuotuose ISI duomenų bazėje); **2** straipsniai kituose recenzuojamuose moksliniuose leidiniuose; **1** straipsnis konferencijų pranešimų medžiagoje.

Tyrimų rezultatai buvo pristatyti ir aptarti **6**-iose nacionalinėse ir tarptautinėse konferencijose.

Disertacijos struktūra

Disertaciją sudaro 5 skyriai, literatūros sąrašas ir priedai.

Pirmajame skyriuje pateikiamas įvadas, problemos aktualumas, tyrimo objektas, darbo tikslai ir uždaviniai, mokslinis, naujumas, praktinė darbo reikšmė.

Antrajame skyriuje aprašomas stochastinio tiesinio programavimo algoritmų analitinis tyrimas.

Bendru atveju STP uždaviniai formuluojami kaip stochastinio dinaminio programavimo uždaviniai. Vienas plačiausiai taikomų ir dažniausiai sprendžiamų STP uždavinių yra dviejų etapų stochastinio tiesinio programavimo uždavinys. Jei duomenų neapibrėžtis šiame uždavinyje yra aprašoma tolydžiuoju tikimybiniu dėsniu, tai jis gali būti suformuluotas kaip netiesinio programavimo su tiesiniais ribojimais uždavinys.

Aptariamos tiesinio ir netiesinio stochastinio programavimo uždavinių formuluotės. Apžvelgiami jiems spręsti dažniausiai taikomi netiesioginiai ir tiesioginiai metodai, Dancigo - Vulfo ir Benderso dekompozicijos metodai.

Apžvelgiami taikomieji stochastinio tiesinio programavimo uždaviniai: transporto valdymas, gamybos valdymas, tiekimo grandžių vadyba ir kt.

Aptariami stochastinio optimizavimo metodai: stochastinio kvazigradiento projektavimo, stochastinės aproksimacijos, Monte Karlo.

Trečiajame skyriuje pristatomas dviejų etapų stochastinio tiesinio programavimo uždavinio sprendimo metodas:

- Monte Karlo imčių generavimas;
- gradientinė paieška;
- projektavimas į leistinąją sritį;
- projektavimas ε-leistinosiomis kryptimis;
- imties tūrio parinkimas;
- statistinių optimalumo kriterijų patikrinimas.

Skyriuje aprašomi antrojo etapo optimizavimo uždavinio sprendimo algoritmai.

Ketvirtajame skyriuje pristatomas dviejų etapų stochastinio tiesinio programavimo uždavinio algoritmų tyrimas.

Nagrinėti keturi stochastinio diferencijavimo metodai:

- tiesioginio diferencijavimo metodas kai diferencijuojama pointegralinė funkcija,
- baigtinių skirtumų metodas,
- modeliuojamojo pokyčio metodas,
- tikėtinumo santykio metodas.

Aprašomas netiesinio stochastinio programavimo uždavinio tikslo funkcijos gradiento tyrimas. Nagrinėta kvadratinių tikslo funkcijų klasė. Pateikiami tyrimo rezultatai.

Aprašomas optimalumo hipotezės kriterijaus patikimumo tyrimas. Nagrinėti dviejų etapų tiesinio stochastinio programavimo uždaviniai, turintis pirmame etape n_1 kintamųjų bei m_1 ribojimų ir antrame etape n_2 kintamųjų bei m_2 ribojimų. Duomenys paimti iš internetinės duomenų bazės adresu http://www.math.bme.hu/~deak/twostage/. (nuoroda kreiptasi 2006-01-20).

Siekiant palyginti skirtingus tiesinio stochastinio programavimo metodus, 1-ojo pavyzdžio uždavinys buvo sprendžiamas Dancigo – Vulfo dekompozicijos metodu bei Simplekso programa.

Pateikti pasiūlytu metodu spręstų taikomųjų uždavinių – darbo organizavimo valdymo bei investicijų ir elektros energijos gavybos planavimo – sprendimo rezultatai.

Penktajame skyriuje pateikiamas gautų rezultatų apibendrinimas, išvados bei rekomendacijos.

Disertacijos pabaigoje pateikiamas literatūros sąrašas ir priedai.

Prieduose pateikiami: Dancigo - Vulfo dekompozicijos metodo taikymo pavyzdys, Benderso dekompozicijos metodo taikymo pavyzdys, duomenų bazės uždaviniai, AMPL taikymas, relaksacijos metodas, kintamųjų atskyrimo metodas.

IŠVADOS

Sprendžiant darbe iškeltus uždavinius yra pasiekti tokie rezultatai:

1. Sudarytas dviejų etapų stochastinio tiesinio programavimo iteracinis algoritmas Monte Karlo metodu.
2. Sudaryti ir ištirti stochastinio diferencijavimo algoritmai:
    - dualaus uždavinio sprendimo,
    - baigtinių skirtumų,
    - modeliuojamojo pokyčio,
    - tikėtinumo santykio.
3. Sudarytas stochastinio gradiento ε-projektavimo algoritmas bei ε-projektavimo algoritmas stochastiniam gradientui reguliarizuoti.
4. Sudaryta ir ištirta Monte Karlo imčių tūrio parinkimo taisyklė, garantuojanti konvergavimą bei racionaliai panaudojanti skaičiuojamuosius išteklius.
5. Sudarytas algoritmas optimalumo hipotezei patikrinti, taikant statistinius kriterijus.
6. Sudarytas algoritmas dviejų etapų stochastinio tiesinio programavimo uždaviniui spręsti naudojant baigtines Monte Karlo imčių sekas.
7. Sukurti algoritmai ir programinių modulių biblioteka Delphi, FreePascal, MathCad, C++ priemonėmis, realizuojanti stochastinio diferencijavimo bei dviejų etapų stochastinio tiesinio programavimo algoritmą Monte Karlo metodu.
8. Sudaryti algoritmai yra pritaikyti taikomiesiems uždaviniams spręsti:
    - personalo organizavimo valdymas;
    - investavimo ir elektros energijos gavybos planavimas.

Remiantis gautais rezultatais bei atliktais tyrimais, galima padaryti tokias išvadas:

1. Sprendžiant testinius bei taikomuosius uždavinius parodyta, jog sudarytas dviejų etapų stochastinio tiesinio programavimo baigtinėmis Monte Karlo imčių serijomis algoritmas leidžia surasti dviejų etapų STP uždavinių sprendinius reikiamu tikslumu, panaudojant priimtinus kompiuterinio laiko ir atminties išteklius.

2. Statistinio modeliavimo rezultatai parodė, kad stochastinio gradiento įvertis, gautas sprendžiant dualųjį uždavinį, leidžia duotu patikimumu patvirtinti hipotezę apie tikslo funkcijos gradiento lygybę nuliui, kai kintamųjų skaičius $n \leq 100$.
3. Sudaryto algoritmo taikymas leido išspręsti visus standartinės duomenų bazės testinius uždavinius bei pagerinti kai kurių uždavinių žinomus sprendimo rezultatus.
4. Statistinio modeliavimo rezultatai patvirtino teorines išvadas apie sukurto metodo konvergavimo greičio eilę. Statistinio eksperimento rezultatai parodė, kad bendra skaičiavimo apimtis, reikalinga uždaviniui išspręsti, yra tiesiai proporcinga skaičiavimų kiekiui, reikalingam įvertinti vieną tikslo funkcijos reikšmę duotu tikslumu.
5. Sudarytas algoritmas nereikalauja papildomų kompiuterio atminties išteklių palyginus su žinomais dekompozicijos metodais, taikomais tokiems uždaviniams spręsti.
6. Sudarytas stochastinių tiesinių uždavinių sprendimo metodas, leidžia gauti tikslesnius sprendinius palyginus su žinomais dekompozicijos metodais, panaudojant mažiau skaičiuojamųjų resursų.

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INVESTIGATION OF STOCHASTIC LINEAR PROGRAMMING BY MONTE CARLO METHOD

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