Science and Arts of Lithuania

LIETUVOS MOKSLAS

THE LITHUANIAN ACADEMY OF SCIENCES
THE LITHUANIAN JOURNAL OF SCIENCE FOR LITHUANIA AND THE WORLD
Published quarterly in Lithuanian and English

'93

Volume I
Book 1

CONTENTS

MATHEMATICS AT THE LITHUANIAN ACADEMY OF SCIENCES

Stasys SKĖRUS

Stasys Skėrus, D. Sc. (Mathematics), scientific secretary of the Institute of Mathematics and Informatics. In 1965-1968 he was a post-graduate student of the Institute of Physics and Mathematics. Since 1968 he worked here as a junior and from 1974 as a senior research fellow. In 1973 he defended his thesis for a Candidate's degree (Physics and Mathematics) on the problems of differential cooperative games. The field of his scientific interests embraces the game theory and other mathematical methods of operations research. Alongside his work at the Institute he runs the courses on optimization methods, operations research, and game theory at Vilnius University. From 1975 to 1979 he was a lecturer at Oran University (Algeria) and since 1992 has been on the staff of the Vytautas Magnus University in Kaunas.

"Each science contains as much science as it contains mathematics"

I.Kant

Even though some interest in mathematics was already made manifest at the old Vilnius University and in the period between the wars a group of highly-qualified mathematicians emerged at the Vytautas Magnus University, systematic mathematical research in Lithuania took start rather late. In the post-war years Professors Petras Katilius, Vytautas Paulauskas, Zigmas Žemaitis, whose students started mathematical research in Lithuania after improving their qualifications at the scientific centres of Russia, worked side-by-side with their younger colleagues. One of them was Jonas Kubilius who graduated from the post-graduate courses at St.Petersburg University and defended his thesis for a Candidate,s degree in physics and mathematics in 1951. In 1952 he began his career at the Institute of Physics and Technology at the newly established sector of physics, mathematics and astronomy. Jonas Kubilius with his works gave rise to mathematical research at the Lithuanian Academy of Sciences. With the application of probabilistic methods in the theory of numbers, in the course of several years he obtained valuable results and pooled them in his Doctor's thesis "Investigations into the Probabilistic Theory of Numbers", defended in 1958. J.Kubilius' monograph "Probabilistic Methods in the Theory of Numbers" (published in 1959) initiated a new branch of mathematical sciences - a probabilistic theory of numbers. Alongside his scientific research J.Kubilius was engaged in extensive organizational activities. On October 1, 1956 the Institute of Physics and Mathematics was founded. Thanks to his efforts the Sector of Mathematics was established at this institute. He was the first to head this sector, simultaneously holding the post of a deputy director for scientific work of the Institute. Due to his active scientific and organizational work J.Kubilius was nominated rector of Vilnius University in 1958. While at the University, J.Kubilius headed the Sector of Mathematics of the Institute for some time and since 1959 this post was taken over by Vytautas Statulevičius, also an alumnus of Vilnius University and a former post-graduate student of St.Petersburg University under the same scientific adviser as of J.Kubilius - Academician Yu.Linnik. However, the trend of V.Statulevičius' scientific work was different - limit theorems of probability theory. During the post-graduate courses and after them V.Statulevičius was in close cooperation with Academician A.Kolmogorov (Moscow), working under him. No wonder, that made a certain influence on the formation of the trend of V.Statulevičius' scientific work. Both his Candidate's thesis "Local Limit Theorems of Nonhomogeneous Markov Chains" (defended in 1959) and his later works open the way for another trend of the Vilnius probability theory school - limit theorems of probability theory. Many of his disciples got involved in these investigations.

As a head of the Sector of Mathematics V.Statulevičius is concerned not only with the development of the probability theory but also with setting up of other branches of mathematical science - a mathematical logic, differential equations, and game theory. The graduates of Vilnius University Vilius Matulis, Eduardas Vilkas, Bronius Grigelionis, Bronius Kvedaras, Mifodijus Sapagovas left for scientific centres of Russia and the Ukraine and upon arrival to Lithuania became the initiators of new trends in mathematical research. Here we shall discuss each of them except for the game theory.

In 1967 V.Statulevičius was elected Director of the Institute

of Physics and Mathematics (previously he was a deputy director).

In the early sixties the work in cybernetics was initiated at the Institute of Physics and Mathematics along with the research in theoretical and experimental physics, mathematics, and astronomy. In 1962 the first computer (BESM-2M) was put into operation at the Institute, in 1964 the Technical Cybernetics Sector was established (head Kostas Žukauskas), and 1965 saw the Light Recognition Processes Sector (head Laimutis Telksnys). On January 1, 1977 after a second reorganization of the Institute of Physics and Mathematics two independent institutes were set up: the Institute of Physics and the Institute of Mathematics and Cybernetics (see Diagram 1). The Institute of Mathematics and Cybernetics brought together the research in mathematics and informatics (according to the present-day classification) as well as in the systems theory. By that time all the trends, now under cultivation at the Institute of Mathematics and Informatics, were almost formed, only the then scientific potential of the Institute could not be equalled to the present one.

Diagram 2 illustrates a structural tree of mathematics at the Institute. The number of researchers of six mathematical departments (as seen at the bottom) totals 83, including 11 Doctors Hab., and 54 Doctors PhD. That is the staff of a reputable research institute. What is the weight of mathematicians of the Institute? Let us compare the numbers obtained with those shown in Diagram 3 (of 1993) and we see that the Departments of Informatics at the Institute include 6 Doctors Hab., 26 Doctors (PhD), the total number of researchers in them being 112, the remaining 63 comprise service, administrative, and technical personnel. Mathematicians make less than a half of the total number of the staff whereas they account for 65% of the scientists (Doctors and Doctors Hab.).

A still numerous body of mathematicians is to be obtained if lecturers of mathematics from all the Lithuanian universities and higher schools are to be considered. More than one of these contains an alumnus of the Institute who has chosen the career of a professor. The Institute (including IFM) has prepared 96 Candidates of Science (PhD) in mathematics, only half of them working at the Institute. Academician V.Statulevičius scores most of them - 23, then follows Acad. E.Vilkas - 12, Acad. B. Grigelionis - 8 (except for postgraduates of Vilnius University). In the course of 20 years (since 1976) 13 mathematicians of the former co-workers of the Institute have defended their theses for a Doctor's of Science (Doctor Hab.) degree. The flourishing of the Institute resulted in a number of generalized works and monographs. In recent years circa ten monographs of mathematics were written and published (by A.Janušauskas, V.Statulevičius and L.Saulis, A.Tempelman, D. Švitra, R.Januškevičius, L.Stupelis), three were issued by foreign publishing houses, five mathematical works won national state prizes.

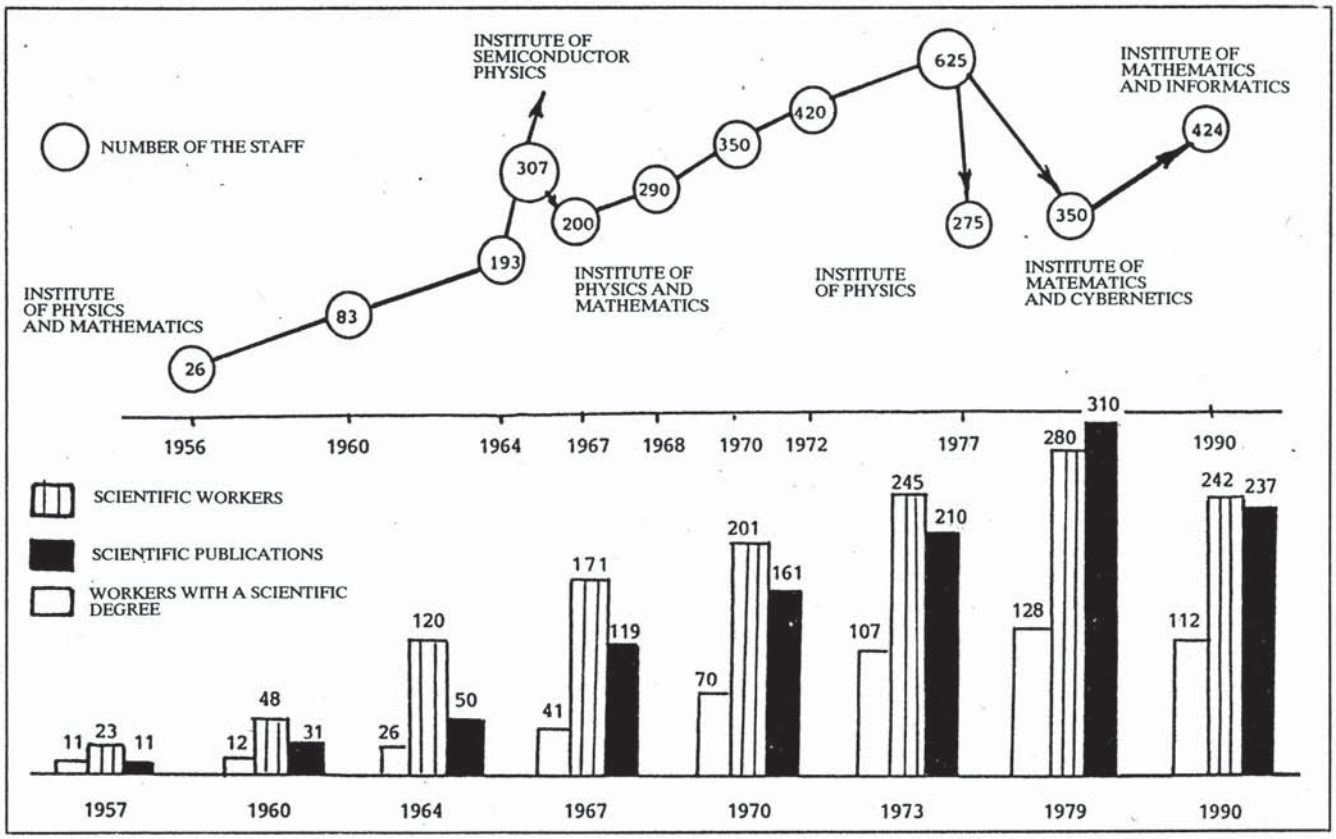
Before starting a survey of scientific achievements of mathematicians of the Institute I should like to note that the results of the game theory and mathematical economics are omitted here since the Operations Research Departament, headed by Acad. E.Vilkas and engaged in this work, was transferred to the Institute of Economics on March 1, 1985. The Ecological Problems Departament (head Dr. D.Švitra) that originated from the Differential Equations Department has moved to the Ecosystems Analysis Centre established in Klaipėda on June 1, 1990, presently belonging to Klaipėda University.

Limit Theorems of Probability Theory

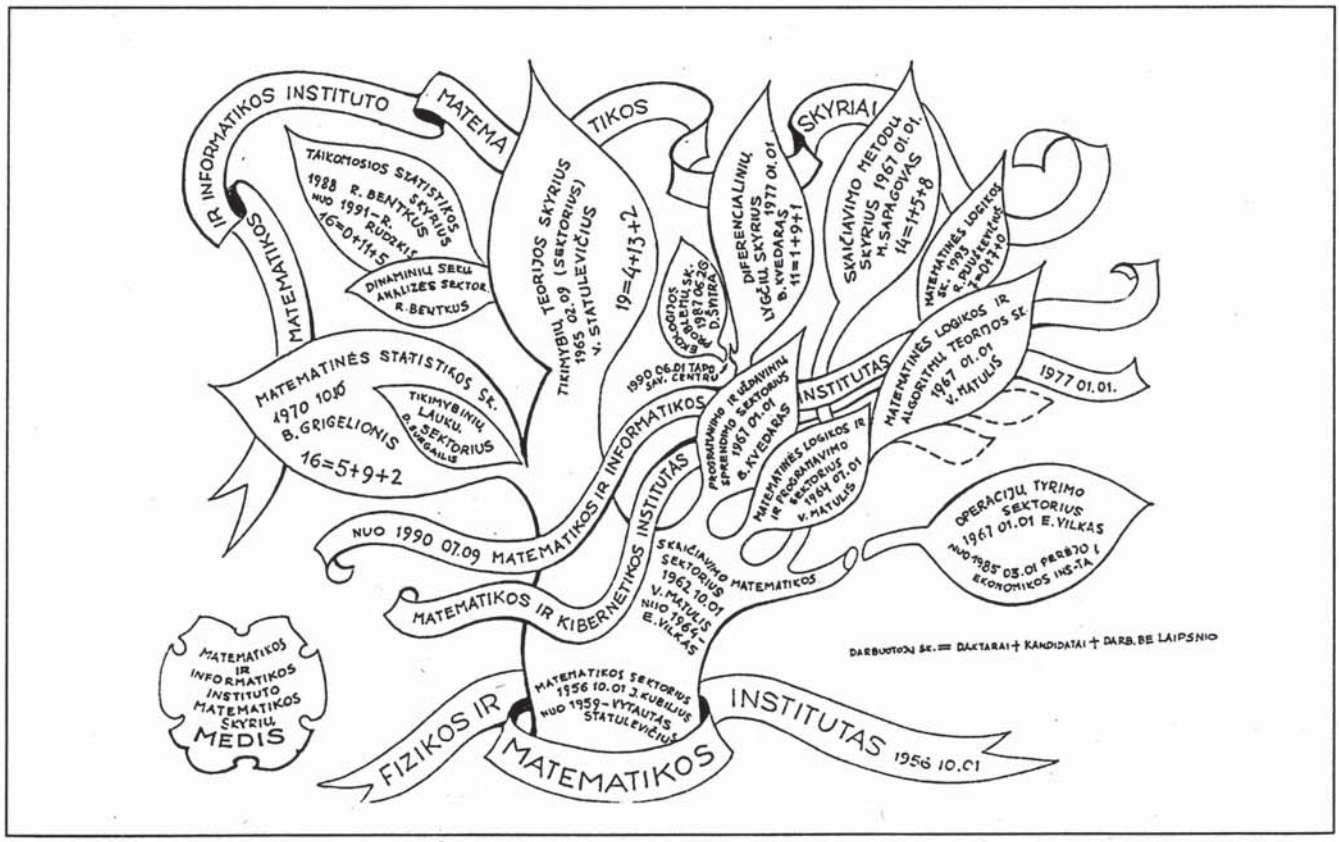
Using the metods of probability theory or mathematical statistics to stimulate various real phenomena, we almost always obtain complex phenomena, frequently expressed by the sums of independent or dependent random variables or integrals of random processes and fields with one or other known characteristics, and sometimes these phenomena are even more complex functionals of initial random processes or fields, for example, solutions of stochastic differential equations. In order to describe these phenomena one must know their probability distribution. Often it is impossible to calculate those distributions even by modern computers, besides, to qualitatively conceive the phenomena we must have their analytical expressions, not obtained by calculations. The objective of the contemporary theory of limit theorems is to approximate these complicated distributions to simpler ones, to determine the accuracy of approximation; if it is insufficient to improve it by adding supplementary members (asymptotic expansions), etc. Expecially subtle is the problem of approximation of small probabilities (local theorems and theorems of large deviations).

All the classics, such as P. Chebyshev, A.Lyapunov, A.Markov, S.Bernstein, A.Kolmogorov, Yu. Linnik, N.Smirnov, A.Khinchin (Russia), E.Borel and P. Levy (France), H.Cramer (Sweden), N.Wiener and J.Neyman (USA), A.Renyi (Hungary), devoted a great part of their works to limit theorems. More and more scientists penetrate into the problems of limit theorems, whereas we represent this subject most plentifully.

As mentioned, the theme of V.Statulevičius' PhD thesis was "Local Limit Theorems for Nonhomogeneous Markov Chains". Similar theorems were proved by A.Kolmogorov and S.Sirazhdinov but only for a homogeneous case: the former did that by reducing to recurrent relations, and the latter by using the spectral theory of characteristic matrices. However, none of these methods suits to a nonhomogeneous chain. Even the first local theorem obtained by V.Statulevi-



Development of the Institute of Mathematics and Informatics; dynamics of its staff and scientific publications.



A drawing by V.Šinkūnienė.

čius was awarded the second prize of Leningrad (at present St.Petersburg) University in 1956 (the first prize went to Acad. V.Fok for the book "Space, Time, and Gravity Theory"). It was for the first time in the history of Leningrad University that a post-graduate student was awarded this prize.

V.Statulevičius has proposed the metod of logarithmic derivatives of characteristic functions which enabled the solutions of this problem to be reached. After V.Statulevičius´ return to Vilnius, A.Raudeliūnas (VU) has generalized his results for a multidimensional case, A.Aleškevičienė for the case of stable limit laws. V.Statulevičius devoted his doctoral thesis to the problems occurring in the passage from the independence to dependence of random variables when making an asymptotic analysis of distributions of the sums of random variables investigating the rate of convergence, asymptotic expansions, probabilities of large deviations.

In collaboration with B.Riauba he has introduced a new class of Markov type processes (though not necessarily Markov) and B.Riauba has proved for these processes local theorems analogous to the Markov processes. It turned out that this new class of processes is important to statistical physics. Considering the asymptotic properties of the sums of random variables related to a complex Markov chain G.Aleškevičius has obtained a great deal of results (including the strong law of large numbers).

In order to go over to the summation theory of dependent variables it was necessary to solve several problems of the sums of independent random variables when random variables are nonuniformly distributed in the main. For the first time V. Statulevičius has obtained asymptotic expansions of Lyapunov´s fractions, general conditions for densities in a local theorem, together with A.Mitalauskas he has presented the most general known so far conditions for lattice random variables in a local theorem. A lot has been done in the investigation of local theorems of densities and lattice random variables, asymptotic expansions, and the rate of convergence by P.Survila, J.Banys, V. Pipiras (now working at the Vilnius Pedagogical University) and by S.Steišūnas. N.Kalinauskaitė has got interesting results in the field of strong limit theorems analysing asymptotic regularities of the sums of random variables and trajectories of stable processes.

Especially many shortages there were in the theory of small probabilities (of the so-called large deviations) in order to apply it to the sums of dependent random variables. There was a lack of general results. V. Statulevičius has filled this gap having proved a general lemma of large deviations, published in the journal "Zeitschrift für Wahrscheinlichkeitstheorie und verwandte Gebiete" in 1966.

Later on L.Saulis, R.Rudzkis, V.Statulevičius have generalized it to the most general cases. The consideration of distributions in multidimensional and infinitely measurable spaces was getting urgent. L.Vilkauskas set about to investigate local theorems of large deviations in multidimensional spaces. Later A.Aleškevičienė and V.Svetulevičienė have done a great deal to solve this problem: they succeeded in proving the theorems of large deviations for a wide class of sets.

In 1967 V.Statulevičius defended his doctoral thesis "Investigations of Limit Theorems of Probability Theory", in which the cumulant method was one of the basic ones. V.Statulevičius together with V.Petrov (St. Petersburg University) and V.Zolotarev (V.Steklov Institute of Mathematics, Moscow) were awarded a Markov prize just established by the USSR Academy of Sciences (1971). In 1979 A.Borovkov (Siberian Branch of the USSR Academy of Sciences), V.Sazonov (V.Steklov Institute of Mathematics) and V.Statulevičius were awarded the USSR State Prize for the development of factorization, convolution and cumulant methods. The cumulant method is further improved, revealing its more and more general possibilities (R. Rudzkis, L. Saulis, V. Statulevičius, R. Bentkus), it is applied in the investigation of sums of weakly dependent random variables (V. Statulevičius, D.Jakimavičius), stochastic integrals (A.Plikusas), polynomial Pittman statistics (A. Basalykas), U-statistics (A.Aleškevičienė), polynomial forms (A. Basalykas, L. Saulis, V.Statulevičius), evaluation of spectral densities (R. Bentkus).

The metrics method for an asymptotic analysis of dependent variables in an infinite dimensional case (P.Gudynas) is being improved as well as the method of logarithmic derivatives to explore the convergence rate in limit theorems of random processes and fields (V.Statulevičius, J.Sunklodas).

A.Aleškevičienė and her disciples V.Naudžiūnienė, L.Griniuvienė, B.Kryžienė, B.Kaminskienė have accomplished a complete asymptotic analysis of renewal processes (V.Liutikas, Vilnius Technical University, and A.Baltrūnas also worked here). Later on distributions of polynomial forms, multiple integrals, various nonlinear statistics have been intensively investigated (V.Statulevičius, R.Bentkus, A.Basalykas, A.Plikusas, L.Saulis, A.Aleškevičienė).

With the extension of applications of the probability theory more and more abstract models were needful. At first the variables were considered on the real line with a further passage to infinite-dimensional functional spaces and still more general mathematical structures.

When investigating the distributions in a group A.Grincevičius has achieved important results having proved the central limit theorem in a solvable matrix group and described the gravitation field of a stable variable in a group of Euclidean space tranformations.

The methods of integral geometry were actual all the time in the theory of geometric probabilities. A disciple of Acad.

V.Prokhorov (Moscow) E.Gečiauskas has explored the distribution function of random cord length of a convex domain as well as the distribution function of the distance between two random points in a convex domain.

The qualifications of young mathematicians went up rapidly, a distinctive school of mathematics was taking shape, more and more often our research works aligned to the front positions of world science. In the 6th-7th decades Berkeley symposia were a gathering of most distinguished world specialists in probability theory and mathematical statistics. Since 1973 the Vilnius international conferences on probability theory and mathematical statistics follow the traditions of the famous Berkeley symposia. Vilnius has become a venue of the most famous forum of probabilists. The conference is held every four years. Five conferences have already been held, the sixth one is to be held this year. The Vilnius probabilists are often invited as speakers of international forums. Vilnius is visited by many guests from foreign scientific centres wishing to make their reports, to work or to discuss with their colleagues.

A.Aleškevičienė defends her doctoral thesis on the topic "Limit Theorems for the Maxima of Sums of Independent Random Variables and Renewal Processes" in which the asymptotic analysis problem of maxima and minima distributions of the sums of random variables has been solved as well as some problems of renewal theory and random walk on a straight line using direct probabilistic and analysis methods. Later on A.Aleškevičienė studied local fields of the processes, asymptotic properties of the distributions of various statistics.

In 1986 L.Saulis defended his doctoral thesis "Approximation by the Normal Law Taking into Account Large Deviations" in which be extended the cumulant method, obtained asymptotic expansions of large deviations and showed that this method was successfully applicable considering the probabilities of large deviations of random vectors. These methods of investigation were efficiently used to explore asymptotic properties of the distributions of nonlinear functionals and dependent random variables.

In his thesis "Asymptotic Analysis of Independent Random Elements in a Banach Space" V.Bentkus has thoroughly analysed a specific character of infinite-dimensional spaces in limit theorems of probability theory. Recall that the first doctoral thesis in Vilnius in this field "Limit Theorems of the Sums of Independent Random Variables in Banach Spaces" has been written by Vyg. Paulauskas (VU), and record estimates of the rate of convergence have been obtained in it. Afterwards V.Bentkus was the first to obtain fundamental results in the investigation of the probabilities of large deviations in Banach spaces.

A new promising trend - the investigation of quantum probability theory - was begun. Using the cumulant method, central limit theorems (CLT) for the case of Gauss and Poisson limit laws have been proved in quantum probability theory and for much more general cases than those considered so far (V.Statulevičius, V.Sidaravičius).

In recent years especially valuable results have been obtained. Optimal results in estimating the rate of convergence have been obtained in the central limit theorem for random field and in the martingale CLT.

Asymptotic expansions for sums of polynomials of random elements with values in general Banach spaces have been obtained both in the central and in local theorem. Asymptotic expansions for distributions functions and their derivatives have been proved. The estimates of residuals obtained are final. The results are applied in general w-statistics.

A long-standing CLT problem for U and V statistics has been solved.

Transition phenomena in a random walk have been explored.

The CLT for polynomial transformations of the Poisson process has been proved in the case of an unbounded kernel function. The approximation of distributions of centered process polynomial forms has been proved by the distributions of multiple integrals with respect to Gaussian measure.

Integral metrics among the elements of random functional spaces have been introduced and studied, and approximate theorems of large deviations have been obtained with their help. The estimate of a characterization stability of the exponential law has been obtained.

The estimates of stability of some dicontinuous distribution decompositions in a uniform metric have been considered, and in the case of the unit law an exact expression of the estimates has been defined.

The existence and uniqueness of the calibrating fields Gibbs measure, constructed according to the classical potential, have been proved and the cumulant method has been applied to noncommutative operators in the CLP of quantum mechanics.

The asymptotic of large deviations of nonlinear statistics and nonlinearly transformed elements of Banach spaces has been explored. The approximation exactness in the CLT has been obtained for the statistics in the multivariate case. Limit theorems of nonlinear transformations have been proved for the processes with independent increments, and expressions of limit laws as stochastic integrals according to Wiener's measure have been found. The conditions have been determined when the k-th order distribution of multilinear form converges to the Hermit polynomial of normal size, and the asymptotic expansion has been obtained.

Necessary and sufficient conditions of converging the distributions of random sums in a continuous metric have been determined. The approximation of the solutions of Wiener-Hopf's equation has been obtained for the

functions having a probabilistic sense.

A theorem of large deviations in the case of the Poisson approximating law has been proved for the sums of dependent random variable series with ψ-mixing (V.Statulevičius). The structure of an asymptotic expansion remainder term of large deviations for a distribution density of the sums of independent random variables has been considered under the regularity condition of Lyapunov fractions (L.Saulis).

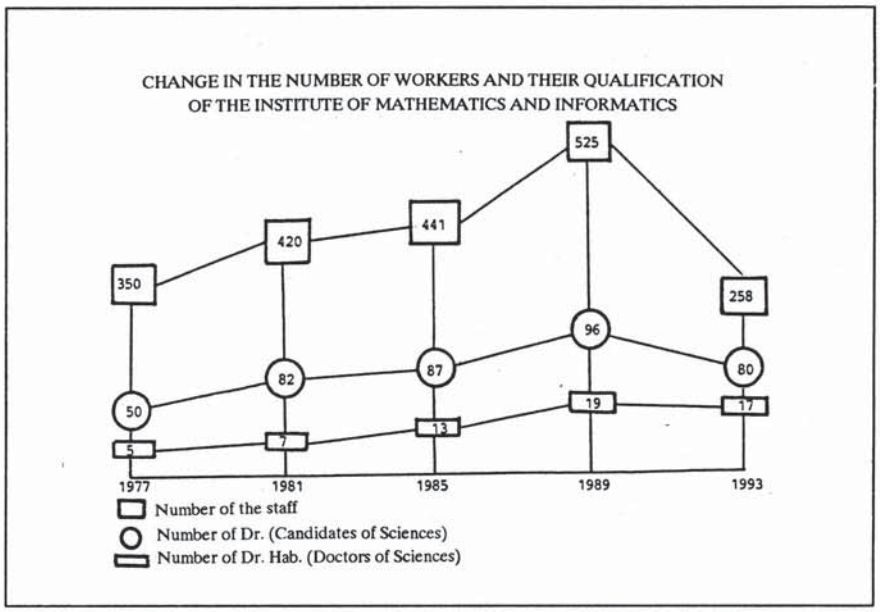
The distributions of multilinear forms were considered further. The functional CLT has been proved as well as the theorem for uniform functionals in C [0, 1] of multilinear of the asymptotic behaviour (A.Basalykas).

Several asymptotic results have been obtained for two classes of U-statistics: the estimates of remainder terms with probability 1 have been obtained, expressed by truncated U-statistics in terms of ordinary U-statistics as well as theorems of large deviations and estimates of the remainder terms in functional limit theorems (A.Aleškevičienė).

All kinds of statistics are considered, for instance, asymptotic expansions for w^{2p} statistics have been proved under very weak weight conditions. The Berry - Esseen estimate has also been proved for multivariate L-statistics (R.Zitikis).

Asymptotic properties of multiple integrals for the processes with independent increments have been considered. The central limit theorem and probabilities of large deviations have been proved (A.Plikusas).

The optimal rate of convergence in the CLT has been established for m-dependent random variables in the case of Abel summation. Analogous estimates have also been obtained for any infinite sum of m-dependent random variables. For ψ-mixing random variables the exponential inequalities of Bernstein, Bennet-Hoeffding, Nagayev-Foucault type have been obtained which include the mixing



Change in the number of the staff of the Institute of Mathematics and Informatics and their qualifications (1977–1993).

The Institute of Mathematics and Informatics was housed in this building in Vilnius in 1985.

Academician Dr. Hab., Prof. Vytautas Statulevičius, Director of the Institute, Head of the Probability Theory Department.

coefficient as a multiplier (J. Sunklodas).

Lately the probabilists of our Institute began active investigations of the problems of noncommutative (or quantum) probability theory in which the application of the cumulant method has good prospects. V. Statulevičius and V.Sidaravičius have proved CLT of Gauss and Poisson laws in the quantum probability theory with the use of the cumulant method for a more general case than that considered so far. During these investigations there was an intense collaboration with mathematicians of Rome and Paris universities.

The Theory of Random Processes

The first works in Lithuania of this trend were written by E. Grigelionis at the beginning of the seventh decade. Later on the work in this field was undertaken by his disciples: Donatas Surgailis, Henrikas Pragarauskas, Remigijus Mikulevičius (Doctors of Physics and Mathematics), Rimas Banys, Rimantas Morkvėnas, Kęstutis Kubilius (Candidates (PhD) of Physics and Mathematics), mathematicians of Vilnius University Doctor of Physics and Mathematics Vygirdas Mackevičius, Candidate of Physics and Mathematics Ričardas Kudžma and others.

In the first works by B.Grigelionis important point processes of queuing and reliability theory were considered, which are the sums of a great number of independent point processes of low intensity. Extending the works of K.Palm (Sweden) and A.Khinshin (Russia), B.Grigelionis has found necessary and sufficient conditions for a weak convergence of such processes to the Poisson processes and investigated asymptotic expansions of their multivariate distributions. These results are widely applied in various fields. Still later the work of this trend was continued by R.Banys.

A rapid development of the theory of random processes was determined by the theory of sequential analysis and decidable functions created by A.Wald (USA) in the 5th-6th decades, by the notion of R.Bellman's (USA) dynamic programming, the martingale theory of J.L. Doob (USA), the theory of K.Ito (Japan) stochastic integration and stochastic equations, and the semigroup theory of linear contractive operators by K.Yoshida (Japan) and E.Hille (USA). They changed essentially the theory of Markov processes created by A. Kolmogorov and gave a start to the contemporary theory of potentials, contemporary mathematical statistics and the theory of optimal control of random processes.

Basing on the works of martingale theory by P.A.Meyer (France), A.Skorokhod (Ukraine) and Japanese mathematicians H.Kunita, M.Motoo, S.Watanabe as well as on those of the theory of stochastic integration and Markov processes, in 1968 B.Grigelionis introduced the notions of locally infinitely divisible processes and their local characteristics, found a martingale characterization and, in particular, conditions of random processes with independent increments, when a locally infinitely divisible process is the solution of the stochastic Ito equation. These conditions were investigated in the works of B.Grigelionis and D.Surgailis in 1971. For the first time the notions of the point measure, compatible with a defined σ-algebra family, and its compensation have been introduced and applied, which generalized the notions of the discontinuity measure of Markov processes and the Levy system. Later a martingale characterization of conditionally Poisson point measures and processes with conditionally independent increments has been obtained.

Using the martingale characterization of probability measures and effective Markovian criteria, general criteria of absolute continuity of probability measures have been obtained as well as formulas of Radon - Nycodym derivatives, which in their turn were applied in the statistics of random processes and information theory. The expression of functionals of locally infinitely divisible processes by stochatic integrals has also been obtained, and extending the ideas of M. Fujisaki (Japan), G.Kallianpur (India) and H.Kunita general stochastic equations of nonlinear filtering have been obtained thereby. D.Surgailis has obtained these equations for nonlinear filtering of Markov processes in another method.

R.Morkvėnas has generalized the works by D.Stroock and S.R.S.Naradhan (USA) and I.Gikhman (Russia) to discontinuous limit processes, using the characterization of such processes obtained by T.Komatsu (Japan).

The general theory on a weak convergence of probability measures in topological spaces created by B.Grigelionis and R.Mikulevičius, is based on the martingale characterization of limit points. It comprises a weak convergence of point measures and semimartingales, functional limit theorems of probabilistic number theory, invariance principles of mathematical statistics, diffusion approximations in queuing theory, a weak convergence of random processes with the values in branching manifolds and the weak convergence of semimartingales with the values in Hilbert spaces to the solutions of stochastic evolutions equations.

R.Mikulevičius has constructed random processes in branching smooth manifolds with a common permeable boundary, has explored their uniqueness conditions and the Markov property. Thereby the theory of random processes in smooth manifolds with bounds satisfying the general boundary conditions of Feller - Wentzell type has been generalized. The well-known results of D.Stroock S.R.S.Naradhan, S. Watanabe, S.Nakao (Japan), N. Portenko, A.Skorokhod (Ukraine), S.Anulova (Russia) and others have been combined into an integral theory. These results were brought together in R.Mikulevičius' Doctor's thesis "On the Solutions of the Martingale

Problem", defended in Vilnius in 1986.

In recent years using the methods of the pseudo-differential operator theory R.Mikulevičius and H.Pragarauskas have determined the existence and uniqueness conditions for weak solutions of stochastic differential equations. These results generalized and improved many previous results of D.Stroock (USA) and Japanese mathematicians T. Komatsu, A.Negoro and M.Tsuchiya.

Considering the existence and smoothness conditions of conditional distribution densities of random processes, B.Grigelionis and R.Mikulevičius have examined the existence, uniqueness, and stability conditions of strong and weak solutions of stochastic evolutions equations of parabolic type. The ideas of earlier works of Russian mathematicians N.-Krylov and L.Rozovski, I.Gyongy (Hungary), of the French mathematicians J.Jacod, J.Memin, E. Pardoux and others were used and extended.

The works of H.Pragarauskas were the first to begin the research on the theory of optimal control of random processes at the Lithuanian Academy of Sciences at the beginning of the 1980ies. The key problems of this theory were finding of the payoff function and of optimal or nearly optimal strategies. In many cases the value functions are the solutions of Bellman equations. Dependent on the structure of controlled processes these equations are nonlinear difference, differential, integral, or integral-differential equations. If one knows the properties of their solutions, then one often succeeds in calculating optimal or nearly optimal strategies either.

In collaboration with N.Krylov Henrikas Pragarauskas has created an optimal control theory of jump diffusions and jump processes in the Euclidean space and its layers. Singular integral differential Bellman equations and their free boundary problem have been investigated by probabilistic methods; approximate methods for their solution grounded. The results are generalized in the doctoral thesis of H.Pragarauskas "The Problems of Optimal Control Theory for Solutions of Stochastic Integral Equations", defended in Vilnius in 1984.

In recent years R.Mikulevičius and H.Pragarauskas have studied boundary value problems of integral differential Bellman equations and general conditions for the existence of optimal strategies by probabilistic and analytic methods. Weak uniqueness of degenerate jump diffusion processes and a classical solvability of boundary value problems of nonlinear integral differential equations have also been established in the works of R.Mikulevičius and H.Pragarauskas. Candidate of Phys.-Math. Sci. Arūnas Baltėnas also worked in this field and generalized H.J.Kuschner's algorithms based on the ideas of weak convergence of random processes.

Of late B.Grigelionis has found general criteria of contiguity for the sequences of probability measures; he has introduced a notion of Helinger measures and expressed them by the formula of Feynman-Kac type. He has proved one-sided and two-sided Lundberg inequalities for risk processes in a random Markov medium. B.Grigelionis has determined necessary and sufficient conditions for a weak convergence of arithmetic processes, constructed, according to additive arithmetic functions of J.Kubilius' type with values in a Hilbert space, to stochastically continuous processes with independent increments.

K.Kubilius has obtained the estimate of the rate of convergence of the functional of a semimartingale sequence to the corresponding functional of a diffusion process of special shape in the Levy-Prokhorov metric and established the dependence of the approximation rate of covergence of the reflected diffusion process on the smoothness of its coefficients.

Cand. of Phys.-Math. Sci. Nijolė Kalinauskaitė considered upper and lower functions of random processes with independent increments, and distribution density expansions of spherically symmetric stable processes. Dr. A. Tempelman and Cand. of Phys.-Math. Sci. Elvyra Senkienė and Remigijus Gylys dealt with random processes in a Hilbert space by linear methods. A. Templeman has considered absolute continuity and singularity conditions for the measures of Gauss-Markov processes. Cand. of Phys.-Math. Sci. Albertas Žalys has found the conditions for the convergence of such processes in probability.

The theory of random fields. Candidates of Phys.-Math. Sci. N.Kalinauskaitė and Aldona Katkauskaitė were the first to write articles on the properties of random fields with independent increments at the beginning of the 1980ties. Later on work in this field was continued by D.Surgailis and his disciples, V.Bentkus and Dr. A.Tempelman. Much was done in the investigation of self-similar and Markov fields, and ergodic properties of fields.

As early as 1941 A.Kolmogorov described all self-similar Gauss processes. In 1962 J.Lamperti (USA) noticed that all possible limit laws of the sums of stationary sequences comprise a class of self-similar laws. On the other hand, the notion of self-similarity was displayed in statistical physics - in the theory of the renormalization group of phase transitions. In the 8th decade many mathematicians began the investigations of selfsimilar processes and fields, their domains of attraction and links with the problems of statistical physics. In the works by D. Surgailis, V.Bentkus and Candidates of Phys.-Math. Sci. Arvydas Astrauskas and Liudas Giraitis a new class of self-similar processes and fields, expressed by stochastic integrals in accordance with the Poisson measure, has been explored, the convergence of the functionals of Gaussian and moving average processes to the self-similar ones has been considered, a notion of Apel's rank of the function has been intro-

duced and its role in the mentioned limit theorems revealed. A part of these results has been generalized in D.Surgailis' doctoral thesis "Linear and Random Fields Subordinate to Them", defended in Vilnius in 1981. Some of them have been obtained by P.Brener and P.Major (Hungary), D.Maruyama and M. Maejima (Japan) almost at the same time. Later on the work of V.Bentkus and D.Surgailis was continued by M.Taggu and Wolpert (USA), and V.Verwaat (Belgium).

Cand. of Phys.-Math. Sci. Edmundas Žalys together with P. Blecher (Russia) considered asymptotic properties of classical models in statistical physics. Using the methods of the renormalization group the existence of phase transitions in the well-known Dayson and Migdal models has been established, and "spin" models in hierarchical lattices considered as well. Lately these lattices drew great attention of physicists and mathematicians, since classical models defined in them can be solved exactly by computers.

After extending functional integration in quantum mechanics and field theory (mostly in the works of USA scientists), in the 8th decade a great interest was taken in Markov fields (Markov fields are random surfaces characterized by a classical dependence of Markov type). D.Surgailis succeeded in getting important results in this field in collaboration with Estonian mathematician T.Arak, they constructed and explored a new class of polygonal Markov fields.

In R. Morkvėnas' works martingale fields on the plane were investigated.

In A. Tempelman's works much attention was paid to the generalization of statistical ergodic Neyman theorems and individual ergodic Birkhoff theorems, which are of importance in statistical mechanics and mathematical statistics. USA mathematicians N. Wiener, N. Dunford, H.R.Pitt, A.P.Calderon were the first to generalize these theorems to homogeneous random fields on multidimensional linear spaces, integer-valued lattices and groups, H.Tempelman has proved a general statistical ergodic theorem on amenable groups showing that the so-called sequences of Folner's sets are statistically approximating. In addition, he has proved that in locally compact groups there exist universal generalized approximating sequences of compact sets. A. Tempelman has obtained ergodicity and mixing criteria for homogeneous Gaussian fields. A. Tempelman and his disciples have proved that sequences of concentric unboundedly increasing balls on the Lobachevsky space and the Lorentz group are individually approximating. That was the first strict proof of the analogue of ergodic Birkhoff's theorem on the Lobachevsky space and the Lorentz group. Afterwards ergodic theorems were generalized applying them to homogeneous random measures groups.

A part of the investigations mentioned is generalized in A. Tempelman's doctoral thesis "Convergence and Validity of Regressive Estimates", defended in Kiev in 1975 as well as in his monograph "Ergodic Theorems on Groups", published in 1986.

In recent years A.Astrauskas has studied asymptotic structures of the basic states of Anderson's model and the mean field model. In the mean field model he has established the relationship between localization theorems of the basic states and the asymptotic behaviour of linear evolution in a random stationary medium, and investigated the intermittence effect and the dependence of this effect on the parameters of the model.

The Works in Applied Statistics

Systematic investigation of spectral analysis of stationary processes were begun at the Institute in 1970.

Raimondas Bentkus, a post-graduate student of V.Statulevičius, considered the asymptotics of the normed error distribution of an empirical spectral function with an unbounded increase in the length of the sample observed. He succeeded to prove that in the case of Gauss processes C-continuous functionals are weakly convergent to the corresponding functionals of a limit process if the spectral density of the stationary process is square integrable. T.Bentkus improved and generalized some results of U.Grenander and M.Rosenblatt, I.Ibragimov, T.Malevich, D.R. Brillinger, and others on spectral functions and statistical estimates of spectral mean. These results are obtained in R.Bentkus' thesis for a Candidate's degree, defended in 1972. R.Bentkus showed that the cumulants of empirical spectral mean of a stationary in a broad sense random process were singular integrals and used this fact to prove the asymptotic normality of spectral estimates. Later these results were generalized to some multilinear forms of stationary processes.

As far as the spectral density is the most interesting spectrum characteristic of applications, since 1975 intensive investigations of the estimates of spectral density were begun. R.Bentkus and post-graduate students guided by him have obtained the theorems of large deviations, exponential inequalities for probabilities that the error of the spectral density estimate may exceed the preassigned quantity, as well as asymptotic distribution expansions for the spectral density estimates of stationary Gaussian procceses. Some of the mentioned results for non-Gaussian processes have been obtained and systematized by R.Rudzkis in his thesis for a Candidate's degree.

Analogous investigations of statistical estimates of non-parametric distribution density were orginated in 1980. Efficient results here were achieved by A.Kazbaras, R.Bentkus' post-graduate student, who systematized these investigations in his thesis for a Candidate's degree, defended in 1982.

In collaboration with professors I.Ibragimov, R.Khasminsky,

M.Pinsker, and their disciples the investigations of minimax nonparametric statistical estimates were begun at the sector of the dynamic sequences analysis. The estimates of the minimax non-parametric spectral density as well as distribution density statistical estimates have been defined asymptotically for some widespread cases of a priori information, when the information is defined by the constraints on the rate of the density derivatives or the density of Fourier transform diminishing. These investigations were an extension of some previous investigations perfomed by F.Hayek, I.Ibragimov, R.Khasminsky, R.H.Farell and other mathematicians in the field of asymptotic mathematical statistics. In the works of V.Statulevičius (junior), A.Kazbaras, and R.Rudzkis new interesting estimates of the so-called called adaptive distribution density and spectral density have been obtained which do not require a detailed a priori information. With the approval of Academician A.Kolmogorov the investigations of experimental nonparametric estimates have also been started at the sector, using computers for this purpose.

R.Bentkus has successfully generalized his investigations in the sphere of non-parametric statistics of stationary processes in his doctoral thesis (1985).

Rimas Rudzkis has obtained the same important results in the field of large deviations of nonstationary Gaussian processes. He applied them successfully to investigate the distribution of maximal deviation of the spectral density estimate. R.Rudzkis has developed efficient methods of an asymptotic analysis of distributions of the functionals, bearing the supremum shape, of non-parametric statistical estimates. He has considered the asymptotics of the distribution of maximal deviation of non-parametric estimates of the probability density and spectral density.

Cand. of Phys.-Math. Sci. Marius Radavičius, when considering statistical estimates of the distribution density, has presented

Seminar of the Probability Theory Department (first row, from the left: Dr. Hab. A.Tempelman, A.Aleškevičienė, V.Bentkus, V.Statulevičius.

The staff of the Applied Statistics Department.

Head of the Differential Equations Department B.Kvedaras (first from the left) discussions with his colleagues doctors of mathematics: A.Grigelionis, J.Macionis, O.Dulkytė, S.Rutkauskas

a new formulation of asymptotic effectiveness, generalizing the effectiveness in the minimax sense that of Bakhadur. He has proved a generalized Hayek inequality, facilitating the analysis of statistical estimates. An asymptotic lower bound of the square risk function for statistical distribution density estimates has been obtained in a certain density class, and a statistical estimate constructed, asymptotically reaching this bound.

The relationship between the asymptotic effectiveness of distribution density parameter estimates and the theorems of large deviations was considered, an asymptotically exact bound of large deviations of the parameter estimates has been obtained.

The deviations in a uniform metric of empirical fields, occurring in non-parametric statistics were considered, and the asymptotics of distribution obtained (R.Rudzkis).

At the Sector of the Dynamic Sequences Analysis, later the Department of Applied Statistics, various applied jobs were undertaken, consultations and advice on the application of statistics offered to biologists, physicians, engineers, and the works of engineering, ecology, medicine, geology, economics were performed on a self-supporting basis. We shall mention some most important of them.

Approximation methods for geometric and temperature system errors of coordinate measuring machines have been offered to the Vilnius Branch of the Machine-Tool Scientific Research Institute.

Mathematical methods, algorithms, and software for filtering of digital records of printed, manuscript, and graphic document images have been worked out for the Scientific Research Institute of Electrography.

Algorithms and programs for statistical data processing have been developed for the oil and gas industrial amalgamation of Udmurtia.

Pursuing fundamental research work in statistics and solving applied problems, a laborious body of scientists, well acquainted with statistical methods and capable of using the software of personal computers, has emerged. Several applied program packages (APP) for statistical data processing by personal computers have been developed at the departament: 1) APP to determine the spectral characteristics of random processes, 2) the APP of non-parametric spectral analysis methods, 3) the package for the analysis of distribution density mixtures, and other applied programs.

The cooperation with Lithuanian physicians is going on. Risk factors and survival indices have been analysed by multi-dimensional statistical methods when examining the data of the Cardiology Laboratory of Vilnius University as well as immunology indices on the data of the Oncological Centre.

In collaboration with the State Statistics Department we carry out the work "The Application of Mathematical Statistics Methods in the reorganization of State Statistics". Its target is to determine the fundamentals of a new structure of state statistics in Lithuania. The work comprises the formation of computer databases, elaboration of information search principles, data processing by mathematical methods, and the development of software for these tasks.

The reconstructed state statistics will help the Lithuanian government and other subjects of national economy to really evaluate the economic situation and predict the future under market conditions.

Differential Equations and the Methods of Their solution

It was decided to begin the research of differential equations at the Institute in 1961. Thanks to the concern of the then head of the Sector of Mathematics V. Statulevičius, M.Sapagovas was sent to the post-graduate courses under the guidance of Prof V. Shamansky to the Institute of Cybernetics in the Ukraine B.Kvedaras to Prof. S.Krein of Voronezh University, and L.Stupelis to Prof. O.Ladyzhenskaya at the Leningrad Branch of the V.Steklov Institute of Mathematics. It was these scientists who were the intiators of the investigations on differential equations at the Institute.

Staying at different scientific centres they worked in different fields of the theory of differential equations. B.Kvedaras considered boundary value problems of ordinary differential equations, M. Sapagovas dealt with the solution of elliptic equations by finite difference methods, and L.Stupelis tackled mixed problems of partial differential equations. The first thesis for a Candidate's degree (PhD) was defended by M.Sapagovas in Kiev in 1965. B.Kvedaras defended his thesis a year later, and L.Stupelis in 1967.

Since 1964 the specialists of differential equations rallied at the Sector of Mathematical Logic and Programming of the Institute. In this sector side-by-side with theoretical investigations employing computers, various problems of national economy, economics, and engineering were also being solved. With the extension of the volume of these works young people were admitted to the Institute, most frequently the graduates of the Faculty of Mathematics of Vilnius University. On January 1, 1967 due to the reorganization of the Institute a Sector of Numerical methods (head M.Sapagovas) was set up which concentrated on the analysis of numerical methods of differential equations, and a Sector of Programming and Problems Solution (head B. Kvedaras), the task of which was to pursue theoretical and applied research work on differential equations and to solve technical and economic problems. Since January 1, 1977, having passed over the solution of practical problems, not related to the application of differential equations, to other subdivisions, the latter was named the Sector of Differential Equations (since March 10, 1980 a department).

Two trends of theoretical investigations were formed in it: the theory of differential equations in functional spaces, also including the theory of elliptic equations, and a mathematical theory on liquid flow. Later one more emerged, - the investigation of mathematical models of biology, medicine, and ecology, described by differential equations.

Many fundamental results have been obtained in all these trends. We shall mention here some of them.

Boundary value problems with non-traditional integral conditions for the systems of first and second order differential equations have been considered. The resolvability conditions of these problems and the questions of solution uniqueness have been determined. Green's function has been constructed by means of which solutions to the problem were expressed (B.Kvedaras). The obtained results allowed the classical theory of boundary value problems to be applied in the investigation of the mentioned problems. A method to form the generalized Green function at the spectrum of a differential operator (Cand. of Phys.-Math. J. Macionis) has been proposed. Basing on the function a wide class of spectrum perturbation problems of differential operators has been investigated, and algorithms to perturb the calculation of real values and real functions have been developed (B.Kvedaras). They are applied in theoretical physics when considering some modifications of Schrodinger's operator. Sufficient and in separate cases necessary conditions of the solution existence for a degenerate differential equation in a Banach space have been determined. The structure of solutions to these equations has been defined and some boundary value problems solved (J. Macionis).

Using the methods developed at the Department of differential equations, weakly degenerate first order differential equations in a Banach space have been considered. The results of these investigations permitted one to apply the Fuks theory, created for ordinary differential equations with analytic coefficients to differential equations in Banach spaces with the operators analytically depending on the argument.

Most distinguished results have been obtained when analysing degenerate elliptic equations. When an equation becomes degenerate or is not strictly elliptic there emerge some effects not allowing to apply the known methods to investigate them. The investigations of such equations were initiated in 1951 with the appearance of M.Keldysh's works. Candidates of Phys.-Math. Sci. D.Jurgaitis, B.Kvedaras, S.Rutkauskas have analysed the structure, asymptotic properties of solutions to strongly degenerate in hyperplane elliptic equations and systems, developed the methods to calculate the members of asymptotic expansion of the solution. They have investigated several non-classical boundary value problems and determined conditions under which classical problems are neither correct nor unsolvable; they have considered wide classes of correct non-classical problems, conditions of their solvability and solution uniqueness, the asymptotics and other properties of solutions. E.Paliokas has considered the systems of elliptic equations weakly related to Cauchy-Rymann multidimensional analogues, and determined the dependence of the Dirichlet problem solvability on the coefficients of the system in various halfspaces and an infinite layer. The obtained results are an important contribution to the analytic theory of degenerate elliptic equations, rapidly developing at the present time, as well as to the theory on boundary value problems of these equations.

L.Stupelis considered second order partial differential equations in the fields in which the type of equations changes. He has explored when the initial and boundary value problems are correct, established the existence and uniqueness of solutions and their qualitative properties. Afterwards he began considering the flow of stationary liquids in a vessel with a free surface, described by a system of Maxwell equations. Reynolds' number being small, he has proved the existence and uniqueness of solutions in weighted spaces. He has also considered several model problems in various weighted functional spaces, determined a priori estimates of solutions. K.Samaitis has investigated boundary value problems in infinite strips and some elementary domains with zero angles for the systems of Navier-Stokes and Maxwell equations.

Very valuable results in the investigation of boundary value problems for the Navier-Stokes system in infinite domains with a free surface were obtained by K.Pileckas. He was the first to find the asymptotics of a free surface at infinity and the dependence of solutions on it. He has investigated the properties of field spaces of solenoidal vectors and determined when the solutions were the elements of these spaces. He has solved several problems in which he considered the motion of viscous two infinite branches and a free surface. A.Grigelionis has obtained necessary optimum conditions for free boundary problems.

Dr. of Phys.-Math. Sci. Donatas Švitra (presently working at Klaipėda University) considered nonlinear differential equations with a retarding argument, when the retardation depends on the function in search. He has made a detailed analysis of the properties of bifurcation problems, especially the properties of their quasipolynomial roots. Some time later D.Švitra set to the research of mathematical models of biology and medicine, described by systems of differential equations with a retarding argument. He and his collaborators under him (A.Grikienis, R.Jančiauskas, R.Laugalys, J.Valančius) have constructed and considered mathematical models of blood production processes, a mathematical theory of thyroid functioning, the problems of mathematical modelling of gluciomia and insulin regulation in blood,

a mathematical model of laboratory population of insects, and other problems.

Cand. of Phys.-Math. Sci. Ona Dulkytė was engaged in the application sphere of differential equations. She has developed a program system to calculate geofiltering parameters according to the general pumping amount. The system is used to estimate exploitation resources of underground water when locating new water reservoirs in Vilnius, Tyumen, Minsk, Moscow, Kharkov, and other places.

The investigations of the theory of differential equations acquired a new impulse upon arrival from Novosibirsk of Dr. Prof. Algimantas Janušauskas. During the last five years five monographs have been written and published on this subject (by A.Janušauskas, D.Švitra, L.Stupelis).

Boundary value problems for strongly degenerate elliptic equations were investigated further. The boundary value problem of Dirichlet type for degenerate equations in a hyperplane has been investigated; in the domain of tubular type the conditions of its solution and solution uniqueness have been found, the maximum principle has been proved and the Dirichlet problem for a system of elliptic equations has been solved in the domain with a singularity at the inner point.

Navier-Stokes equations were considered in the domains with two infinite branches and a free surface. The theorems of solution existence and uniqueness for them have been obtained as well as the asymptotics of solutions and the free surface.

The Riemann-Hillbert boundary value problem for system of differential equations has been investigated, which are three- and four- dimensional analogues of the Cauchy-Riemann system. The criteria have been found when this problem is Noetherian.

Some models of the flow of viscous incompressible liquid have been explored.

Along with the research work of differential equations the work on solution methods of these equations was pursued. This trend of research became the key subject of the Numerical Analysis Department from the very establishment of this department in 1967.

Real phenomena and processes of physics, mechanics, engineering, chemistry, biology are described by differential equations. These phenomena are mostly nonlinear, and the corresponding to them nonlinear differential equations can usually be solved only by approximate numerical methods. The solution of such nonlinear differential equations of elliptic type by difference methods made the basis of M.Sapagovas´ thesis for a Candidate´s degree, and afterwards it was one of the key scientific trends of the Numerical Analysis Department for a long time. Dealing with the solution of these equations, approximate methods were newly theoretically grounded, basing on the theory of monotonic operators. Later such a method of solving nonlinear problems was widely considered and extended in the works of many foreign mathematicians.

In the thesis for a Candidate´s degree defended in 1973 and later papers Vytautas Kleiza has considered the solution of nonlinear problems (including elliptic differential equations) by the Monte-Carlo method. He has proved that it was possible not only to calculate an approximate solution of a nonlinear problem by this method but also to numerically verify the criterion of the solution existence and uniqueness of the nonlinear problem. The year 1967 saw the investigations of a new class of differential equations - the problems of real values of differential operators. Difference methods of high accuracy for calculating real values of elliptic differential operators have been developed and substantiated theoretically. For the first time we succeeded to ground the difference methods of higher accuracy for calculating the real values of differential operators with varying coefficients. These theoretical investigations were closely connected with the solution of practical problems (optimization of waveguide parameters by their eigen oscillations). In 1976 Danguolė Sapagovienė defended her thesis for a Candidate´s degree in the field which corresponded to a new trend of computational mathematics under formation at that time, later called a numerical experiment. Within the latest decade the numerical experiment was widely and rapidly developed as a new method for solving scientific technical problems as well as for scientific research of various branches of science.

Since 1972 after the scientific probation of head of the department M.Sapagovas at the universities of Great Britain the investigations of higher accuracy difference schemes for nonlinear differential equations of elliptic type were undertaken at the Department of Numerical Analysis. Two important results have been obtained after some investigations. In principle a new approach of developing and grounding difference methods has been found for quasi-linear differential equations of elliptic type. Symmetrized difference schemes of higher accuracy have been formed and based for weakly nonlinear elliptic equations. This result has been generalized in the thesis for a Candidate´s degree of Rimantas Skirmantas, defended in 1981. Here the numerical methods of solution have been first applied to a new class of problems - microelectronic problems.

Since 1976 in co-operation with a research sector "Vibration Technology" of the Kaunas Polytechnical Institute (KPI) the problems related to a design of contacts from liquid metal were being solved at the Department of Numerical Analysis. That promoted new theoretical investigations of the methods for solving differential equations with an integral condition. Now this trend is one the major theoretical trends. In 1984 Raimondas Čiegis defended his thesis for a Cand. of Phys.-Math. Sci. on this subject, in which he has ex-

haustively considered the methods for solving parabolic equations with an integral condition. These subjects comprised the essential part of a doctoral thesis of M.Sapagovas (1986).

Vytautas Būda has sequentially used the numerical experiment in the problems of nonlinear diffusion as in one of the most important microelectronics technologies in his works, generalized in his thesis for a Candidate's degree in 1986. It has been shown by the numerical experiment method that under certain conditions in a nonlinear diffusion process there may exist an unusual phenomenon of physics - a localization effect which can be successfully used in technological processes. When solving this problem of microelectronics there was a close cooperation with the scientists of the M.Keldysh Institute of Applied Mathematics, such as Academician A.Samarsky, Corresponding member S.Kurdiumov, Dr. Popov (Moscow).

A close connection between theory and applications is characteristic of theoretical investigations in computational mathematics. One of the forms of such a connection is a development of software to solve differential equations. By common effort of the whole body of the Numerical Analysis Department an applied program package was developed in 1979-1981 for solving nonlinear elliptic differential equations. As compared to other analogous works this package bears many advantages. It includes a special high level input language, many modern methods of solving nonlinear differential equations are realized here, it allows the method to be automatically selected. The coworkers of the department have developed over 29 programs, formed on the basis of original solution methods, meant for solving nonlinear differential equations.

In 1985 the consideration of the methods for solving the problems of nonlinear laser optics started (R.Čiegis, S.Norvaišas). Some original results have been obtained when solving nonclassi-

In 1986 four mathematicians of the Institute defended doctoral hab. theses: head of Mathematical Statistics Department prof. B.Grigelionis (in the centre) and Dr. Hab. (from the left): M.Sapagovas, V.Bentkus, R.Mikulevičius and L. Saulis.

On June 28 - July 3, 1993 the Vilnius Sixth International Conference on Probability Theory and Mathematical Statistics was held in Vilnius. It is the major forum of the world specialists of this branch of science. It was overtaken from Berkely (USA, California) in 1977 and takes place in Vilnius every four years. This year it was attended by 180 foreign participants and over 50 Lithuanian mathematicians. Most numerous was the Russian delegation - 69, followed by the Ukraine - 23, the USA - 11, Germany - 10, France and Hungary - 7 (each), Great Britain and Poland - 6 scientists (each). Even representatives of the far off countries, such as Australia, Singapore, Japan, China took part.

The conference was opened with the speech of welcome by President of Lithuania A.Brazauskas. One of the major sponsors, President of the Tauras Bank and Lietverslas Corporation G.Konopliovas started the conference.

The plenary session contained two reports: Professor of Seatle University (USA) J.A.Wellner tackled the problems of statistics, and Professor of Rome Universitty, Vito Volterra Centre of Mathematics, L.Accardi dealt with the problems of the quantum probability theory.

The work went on in 6 sections: limit theorems, stochastic analysis and stochastic physics, statistics, probabilistic number theory, quantum probability theory, and change detection in random processes. 216 reports have been delivered in them. 45-minute lectures were delivered by the most eminent specialists: R.M.Dudley (USA), L.Birge (France), S.Aivazyan, A.Borovkov, R.Dobrushin (Russia), K.D.Elworthy (England), M.Fukushima (Japan), P.Major (Hungary), F.Goetze, W.Shwarz (Germany), J.Kubilius, V.Statulevičius, B.Grigelionis, V.Paulauskas, H.Pragarauskas, D.Surgailis, L.Telksnys (Lithuania) and others.

On the photo a group of participants at the Vilnius Sixth International Conference on Probability Theory and Mathematical Statistics.

Photos by G.Šimkūnas

cal problems of the elasticity theory (Z.Vasiliauskas). The application of the methods for solving the equations of minimal surface in construction mechanics has been extensively considered (G.Kairytė, M.Sapagovas). New efficient methods to calculate the potential of the magnetic field have been created (T.Šeibakas). Together with the evening faculty of the Panevėžys Branch of KPI a mathematical model of thin metallic films has been considered (V.Kleiza, T.Veidaitė). Since 1980 together with the Chair of Physics of the Vilnius Civil Engineering Institute (now Vilnius Technical University) mathematical models for determining and prognosing radioactive pollution of the Baltic sea have been considered at the Institute (V.Kleiza). A lot of other problems, described by differential equations actual for Lithuanian national economy, are being solved.

Efficient numerical methods to solve a system of nonlinear optics differential equations have been developed and substantiated lately.

The asymptotic stability of economical difference schemes to solve multidimensional nonstationary problems has been studied (R.Čiegis).

The solution existence and the number of solutions of ordinary differential equations with a non-local condition has been investigated when the unique solution existence condition is not satisfield, difference schemes to solve these problems have been studied (M.Sapagovas).

Developing the finite difference theory and numerical experiment methods, the convergence of algorithms have been proved for diffusion-reaction problems as well as for diffusion systems of degenerate equations with a nonlinear coefficient of degeneracy. A new class of iteration processes for systems of linear equations has been studied.

A difference scheme of symmetric movement in the magnetic field of viscous one-dimensional barotropic gas has been constructed and investigated, the law of entropy non decrease, valid for heat conductive gas, has been proved.

Mathematical Logic

The initiatior of this trend at the Lithunanian Academy of Sciences was Cand. of Phys.-Math. Sci. Vilius Matulis who defended his thesis in 1964.

Since 1967 the research work in mathematical logic has been pursued at the Department of Mathematical Logic and Algorithm Theory (until 1993 its head was V.Matulis), since 1993 at the Mathematical Logic Department (headed by Cand. of Phys.-Math. Sci. Regimantas Pliuškevičius). The major trends of scientific research were the logical proof theory, estimation of algorithm complexity, and application of the mathematical logic in informatics.

One of the essential rules of logical calculi is the cut known among mathematicians as the inventor's paradox. Its essence is like this: in order to prove some statement, it is often easier and more convenient to prove a stronger statement. In 1935 G. Gentzen (Germany) obtained a fundamental result, - he has found a possibility of eliminating the cut rule from the classical and constructive predicate calculus. This achievement encouraged to study the possibilities of eliminating other structural rules. Such a result in the classical predicate calculus was obtained by V.Matulis, and simultaneously independent of him by a well known Swedish mathematician S.Kanger. In addition, V.Matulis has proved that in the classical predicate calculus one may construct the proof search uniquely, generalizing thereto Wang Hao's (USA) results in the classical propositional calculus.

In the middle of the 7th decade R.Pliuškevičius investigated the admissibility of structural rules in the calculi of constructive logic. Somewhat weaker results simultaneously with and independently of R.Pliuškevičius were obtained by H.Curry (USA). Besides, R.Pliuškevičius has constructed a system of Kanger type for applied calculi of constructive logic. Cand. of Phys.-Math. Sci. Aida Pliuškevičienė has developed a general method how to construct a calculus for the axiomatic theory with equality under very restricted cut type rules. The method was applied in concrete mathematical theories, for instance, a cut-free calculus has been constructed for R.M.Robinson's arithmetic system. Basing on the developed sequential calculi the inverse Maslov method has been generalized in axiomatic theories with equality.

In 1976 R.Pliuškevičius initiated the investigations of logics of programs (algorithmic logics) at the Institute. Logics of programs enable us to make a program analysis and synthesis by the methods of mathematical logic, to define the semantics of various programming languages. It turned out that logics of programs are very substantional nonclassical logics of a new type, containing the elements of algorithm theory and modal logics. Some problems of the proof theory of logics of programs have been studied. Cand. of Phys.-Math. Sci. Jūratė Sakalauskaitė has proved the completeness of a propositional dynamic logic (a non-classical logic introduced by W.Pratt (USA) in 1976 allowing to conveniently describe program properties) with an infinite iteration, thus asserting the famous hypothesis of R.Street (USA). Cand. of Phys.-Math. Sci. Kastytis Gečas has proposed and grounded logical analysis methods of distributed programs, allowing to describe and study the semantic properties of the distribute programming languages.

In 1979 the investigations of algorithm theory were begun. Cand. of Phys.-Math. Sci. Stasys Jukna under the guidance of prominent mathematician Y.Yanov (Moscow) has developed the methods for the synthesis of "reliable" programs from "unreliable" elements, allowing not to asymptotically increase the program complexity even with an increase of program faults. Asymptotic estimates of the corresponding Shenon functions have been obtained which proved the optimality of the methods

proposed. Some time later S. Jukna revealed a general principle of "entropy preservation", and basing on it, developed a method for solving one of the major algorithm theory problems - the construction of efficient complexity estimates. Cand. of Phys.-Math. Sci. Valentinas Kriaučiukas guided by A.Slisenko (St.Petersburg) has described and investigated the classes of search problems in which decompositional methods of solution were efficiently applied.

In recent years many interesting results of the mathematical logic have been obtained. The methods to prove the theorem of the cut rule elimination have been developed for functional dynamic and temporal logics, rather important in informatics. The method of infinitary rule reduction has been presented, which enables one to prove the completeness of a wide class of functional temporal logics. The method of indices has been extended for modal logics. The methods of non-disjunctive resolution of dynamic and temporal logics has been proposed. Effective axiom systems of various non-classical logics, describing the properties of algorithmic processes, have been formed. Efficient methods of specialization of the derivation in various temporal logics have been worked out. On their basis the method for verification of computer network protocols has been developed.

The method of a formal logic description of the process interaction in distributed programs has been proposed. A compositional Hoare type axiomatic system of the distributed processes has also been proposed and proved. The relationship between a dynamic logic and the semantic of logic programming was considered. The Diophantine characterization of NP-complete sets has been obtained simultaneously with and independently of C. Kent and B.Hudson (Canada). A general principle of "entropy retaining" in calculation processes has been proposed. Basing on this principle a general method for constructing lower complexity estimates of logical schemes has been created. A new method to obtain lower estimates has been proposed, based on the instability of minimal schemes. A general criterion of the existence of the universal function in admissible sets has been obtained. The structures without an effective universal function are described.

Since the scientific research results of IMI mathematicians were presented in comparison with the works of scientists from other countries we may judge about the international recognition of our mathematicians. The fact that the most distinguished mathematicians of our Institute V.Statulevičius, B.Grigelionis, D.Surgailis, H.Pragarauskas, M. Sapagovas, etc. were invited lecturers in many international conferences and symposia confirms that, too. Academician V.Statulevičius, the only one of the Lithuanian mathematicians, had the honour to deliver an invited report at the Congress of World Mathematicians in Vancouver (Canada).

Mathematicians of the Institute are invited to the universities or scientific centres of West Europe or America to lecture or pursue research work for a shorter or longer period. Dr. Hab. V.Bentkus, Doctors K.Pileckas, L.Giraitis, and S.Jukna have been granted the scholarships of Alexander von Humboldt and are working at the universities of Germany. Dr. Hab. R.Mikulevičius works as a visiting professor at the University of southern California, Dr. Hab. A.Tempelman at the University of Pennsilvania, Dr. Hab. D.Surgailis is invited to Cleveland University (USA). The largest forum of world probability proficients witnesses world recognition of the Lithuanian school of probability theory.

Recalling I.Kant's thought on mathematics not in so strict a form as he had expressed, we must acknowledge that the application of mathematics is decisive for the level of many sciences. So the sciences of physics, engineering, economics most often cannot do without mathematics. And if we have in mind not only the trend of mathematical science which was reviewed here, but also the sphere of mathematical sciences which along with mathematics embraces informatics, logics, systems investigation, and statistics, then we see that the results of mathematical sciences are also employed in contemporary biology, medicine, social and other sciences.

It goes without saying that the level of science and the proficiency of specialists available make a tremendous effect on the preparation of specialists in each field. Therefore, in view of the future of Lithuania, its intellectual potential we must foster this small but rather strong branch of science - mathematics, - once called by Acad. J.Kubilius "a servant of all sciences".

List of monographs and manuals, written and published by MII scientists

1. *Kvedaras B., Sapagovas M.* Skaičiavimo metodai. – V.: Mintis, 1974; 2. *Bulota K., Survila P.* Algebra ir skaičių teorija. – V.: Mokslas, 1976-1977. – 2 t.; 3. *Колесов Ю.С., Швитра Д.И.* Автоколебания в системах с запаздыванием. - Вильнюс: Мокслас, 1979; 4. *Темпельман А.А.* Эргодические теоремы на группах. - В.: Мокслас, 1986; 5. *Саулис Л., Статулявичюс В.* Предельные теоремы о больших уклонениях. - В.: Мокслас, 1989; 6. *Швитра Д.И.* Динамика физиологических систем. - В.: Мокслас, 1989; 7. *Yanushauskas A.* The oblique derivative problem of potential theory. – New York; London: Consultants Bureau, 1989; 8. *Янушаускас А.* Многомерные эллиптические системы с переменными коэффициентами. - В.: Мокслас, 1990; 9. *Янушаускас А.* Методы потенциала в теории эллиптических уравнений. - В.: Мокслас, 1990; 10. *Saulis L., Statulevičius V.* Limit theorems for large deviations. – Dordrecht; Boston; London: Kluwer Academic Publishers, 1991; 11. *Янушкявичюс Р.В.* Устойчивость характеризаций вероятностных распределений. - В.: Мокслас, 1991; 12. *Бенткус В., Гетце Ф.* (ФРГ), *Гудинас П., Паулаускас В.* (ВУ), *Петров В.В.* (Россия), *Рачкаускас А.* (ВУ), *Сунклодас Й., Саулис Л., Статулявичюс В.* Итоги науки и техники. Современные проблемы математики. Фундаментальные направления. Т. 81: Теория вероятностей - 6. Предельные теоремы теории вероятностей. - М.: ВИНИТИ, 1991. [1993 m. numatoma išleisti angliška knygos vertimą: Encyclopedia of Mathematical Sciences. Springer Verlag (VFR)]; 13. *Ступялис Л.* Краевые задачи для уравнений Навье - Стокса. - В.: Мокслас, 1992; 14. *Tempelman A.* Ergodic Theorems for group actions. – Dordrecht: Kluwer Academic Publishers, 1992.

1993. 07. 10