Comparative Analysis of Homogeneity Tests For Censored Samples Under Crossing of Survival Functions

Gintarė Klimantavičiūtė, Assoc. Prof. Dr. Rūta Levulienė

Vilnius University

Introduction

Survival analysis is a statistical method used to analyze data that examines the time of a specific event. In this study, a power simulation study was conducted to compare homogeneity criteria for censored samples when the survival functions may intersect. These criteria include the log-rank (LR)[2], two-stage procedure (TSPV), proposed by Qiu and Sheng [3], modified log-rank (MLR), and modified informative criterion (MS) proposed by Bagdonavičius et al. [1]. Modeling was performed using various sample sizes and different distribution functions, covering various scenarios where survival functions do not intersect, intersect at the beginning, middle, and end of the time interval.

Homogeneity Tests Used In The Study

Test	Test Statistic
Log-rank (LR)	$\chi^2_{log} = \frac{(O_j - E_j)^2}{Var(O_j - E_j)}$
Modified Score (MS) Modified log-rank (MLR)	$\chi^2 = \mathbf{V}^T \hat{\Sigma}^{-1} \mathbf{V}$
Two-Stage procedure (TSPV)	$V = \sup_{D_\varepsilon \leqslant m \leqslant D - D_\varepsilon} (V_m)$

Power Simulation Study Design

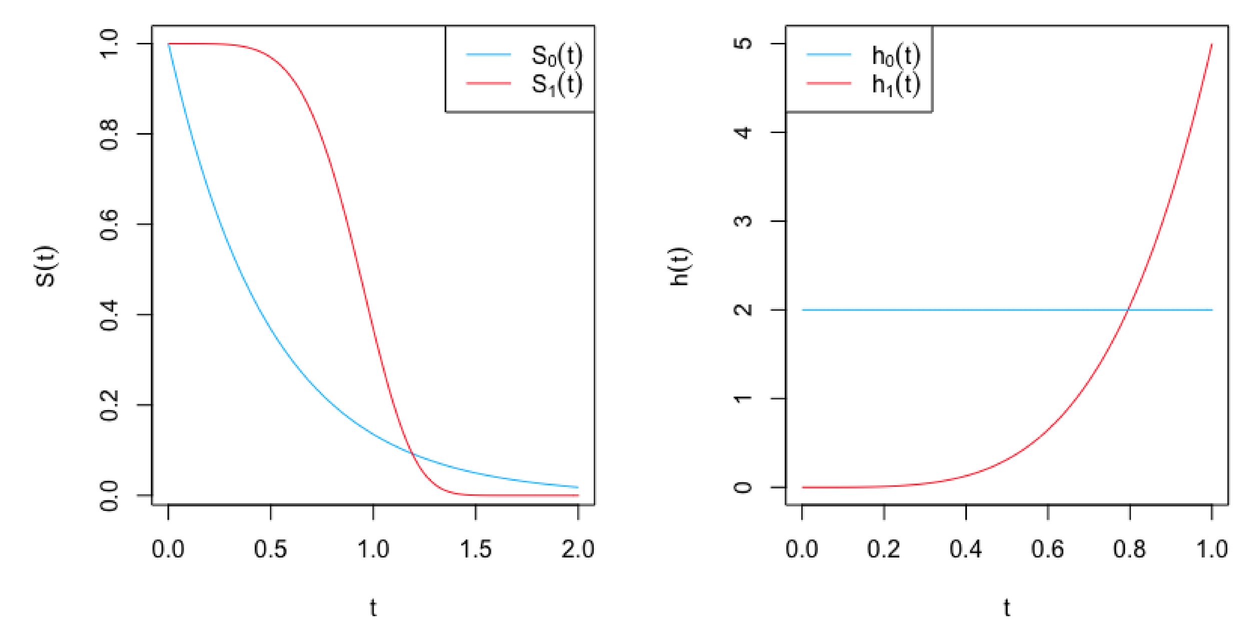
Let $S_0(t)$ and $S_1(t)$ be survival functions of objects in the first and second groups respectively, and let $t \in [0, \tau]$ be the time of interest. Then the hypothesis is $H_0 : S_0 = S_1, H_a : S_0 \neq S_1$. For power computation, data is modelled under alternative hypothesis with following parameters:

- 38 simulated scenarios;
- $N = 1000$ iterations;
- sample sizes per each group were set to $n_i = \{25, 50, 100, 200, 300, 400, 500, 600, 700, 800, 900, 1000\}, i = 0, 1, n_0 = n_1$;
- $5\%, 10\%$ or no censoring was applied $\forall\, n_i$.

Simulated Scenarios

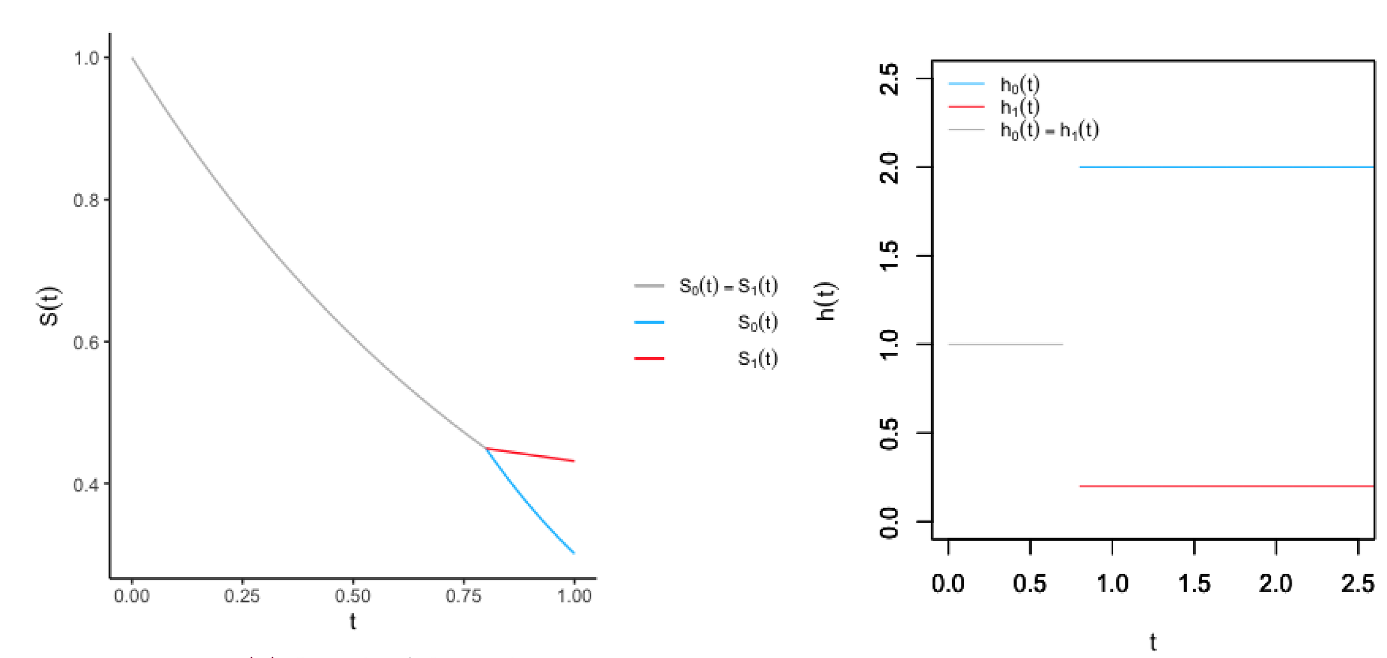
- **Case 1** Survival functions crossing at $t \approx 1.19$.
- **Case 2.** Survival functions cross at $t = 1$.
- **Case 3.** Hazard rates are constant. Survival functions do not intersect.
- **Case 4.** Hazard and survival functions do not cross but differ for large t.
- **Case 5.** Hazard rates do not cross. S_1 experience a downward jump at $t = 0.2$
- **Cases 6.1 - 6.5.** Survival functions following Weibull distribution having shape parameters $\nu_0 = 1, \nu_1 = \{1.1, 1.5, 2, 3, 5\}$ and location parameters $\theta_0 = \theta_1 = 1$.
- **Cases 7.1-7.5.** Survival functions following log-logistic distribution with $\nu_0 = 1, \nu_1 = \{1.1, 1.5, 2, 3, 5\}, \theta_0 = \theta_1 = 1$.
- **Cases 8.1-8.5.** Survival functions following log-normal distribution with $\nu_0 = 1, \nu_1 = \{1.1, 1.5, 2, 3, 5\}, \theta_0 = \theta_1 = 1$.
- **Cases 9.1-9.6.** Survival functions following Weibull distribution with $\nu_0 = 1$, $\nu_1 = \{1, 1.1, 1.5, 2, 3, 5\}, \theta_0 = 0.5, \theta_1 = 1$.
- **Cases 10.1-10.6.** Survival functions following log-logistic distribution with $\nu_0 = 1, \nu_1 = \{1, 1.1, 1.5, 2, 3, 5\}, \theta_0 = 0.5, \theta_1 = 1$.
- **Cases 11.1-11.6.** Survival functions following log-normal with $\nu_0 = 1$, $\nu_1 = \{1, 1.1, 1.5, 2, 3, 5\}, \theta_0 = 0.5, \theta_1 = 1$.

Illustrative Samples of Modeled Data



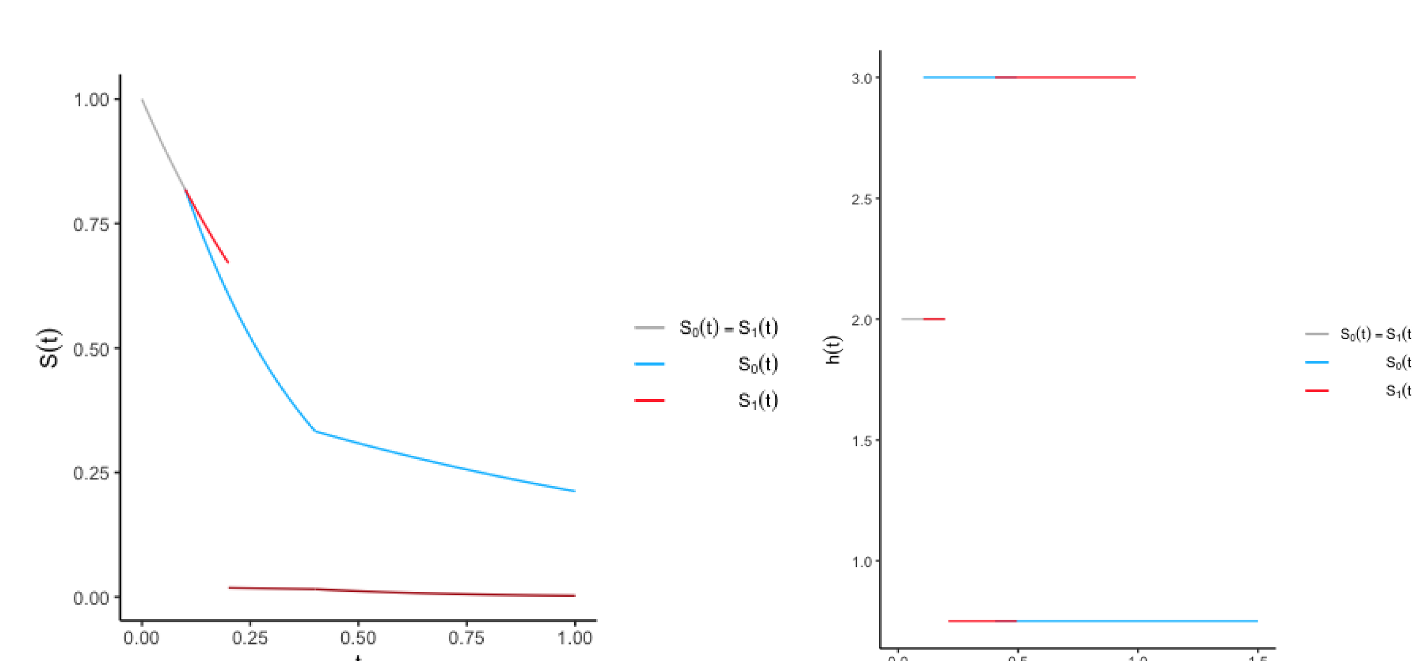
(a) Survival functions (b) Hazard functions

Figure 1. Case 2. Survival functions cross at $t = 1$.



(a) Survival functions. (b) Hazard functions.

Figure 2. Case 4. Hazard and survival functions do not cross but differ for large t.

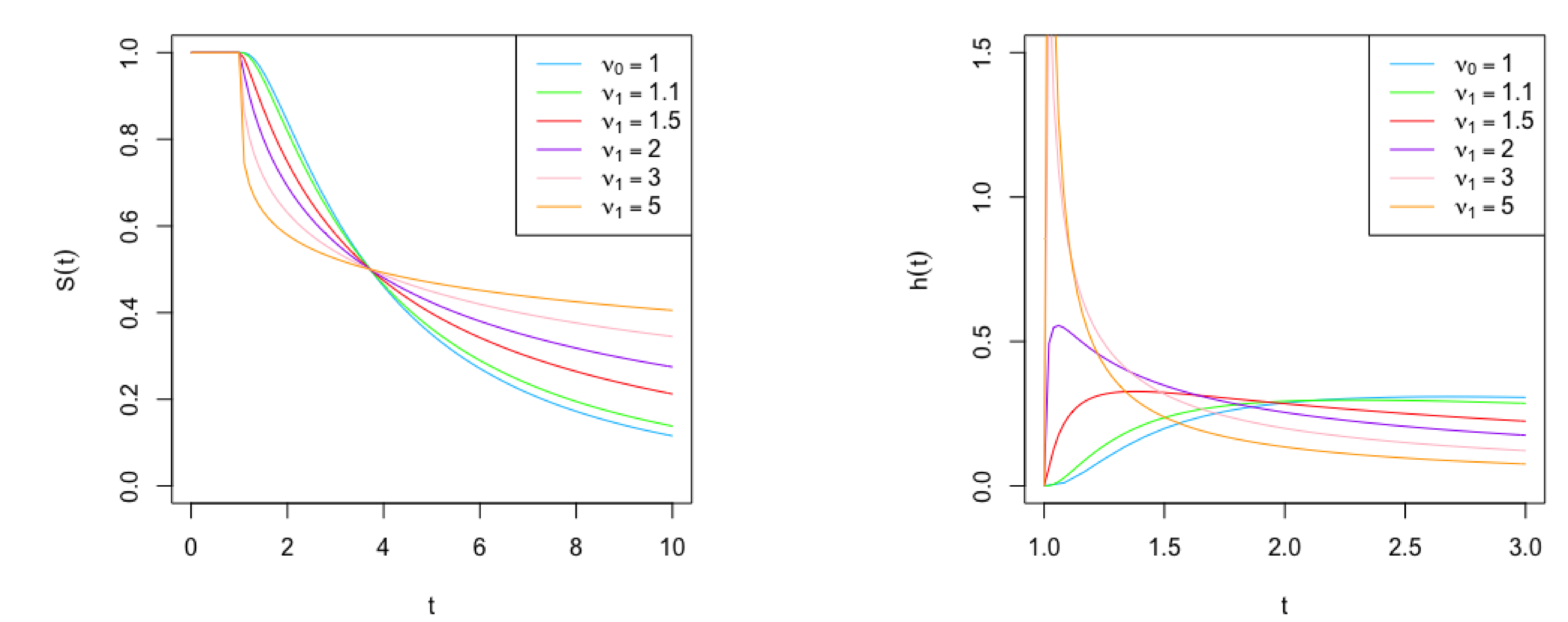


(a) Survival functions. (b) Hazard functions.

Figure 3. Case 5. Hazard rates do not cross. S_1 experience a downward jump at $t = 0.2$

Results

For the situation of the late crossing of survival curves following the exponential distribution, MLR and TSPV test could be a suggested approach in testing the homogeneity of survival curves. For the situation of hazard rates being constant and parallel to one another, MLR might be recommended, especially having small sample sizes. For the situation of survival functions being the same and then differing for large t, MLR and TSPV methods should be considered. MS test might not be recommended when having survival curves with applied censorship that follow Weibull distribution and location parameters for both functions are the same. As well as, when survival functions follow log-logistic distribution having different location parameters. Especially, if the sample size is small (up to 100 per group). For the situation of survival curves following log-normal or log-logistic distributions and having set unequal location parameters, MLR test is recommended as it was the most likely to detect an effect correctly for the smallest censored and uncensored sample sizes. For the situation of survival curves following Weibull distribution, especially if location parameters for both groups are different, TSPV test might be a good recommendation.



(a) Survival functions following log-normal distribution. (b) Hazard functions following log-normal distribution.

Figure 4. Case 8.1-8.5. Survival functions following log-normal distribution having shape parameters $\nu_0 = 1, \nu_1 = \{1.1, 1.5, 2, 3, 5\}$ and location parameters $\theta_0 = \theta_1 = 1$.

Conclusions

The analysis results show that the power of the criteria depended on the specific characteristics of the simulated data, but it was found that the MLR and TSPV criteria performed best in various scenarios. The results provide recommendations to researchers on which statistical method to use when comparing survival curves in censored samples.

References

[1] Vilijandas B Bagdonavičius, Ruta J Levuliene, Mikhail S Nikulin, and Olga Zdorova-Cheminade. Tests for equality of survival distributions against non-location alternatives. *Lifetime data analysis*, 10(4):445–460, 2004.

[2] Nathan Mantel. Ranking procedures for arbitrarily restricted observation. *Biometrics*, pages 65–78, 1967.

[3] Peihua Qiu and Jun Sheng. A two-stage procedure for comparing hazard rate functions. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, 70(1):191–208, 2008.

gintare.klimantaviciute@mif.stud.vu.lt